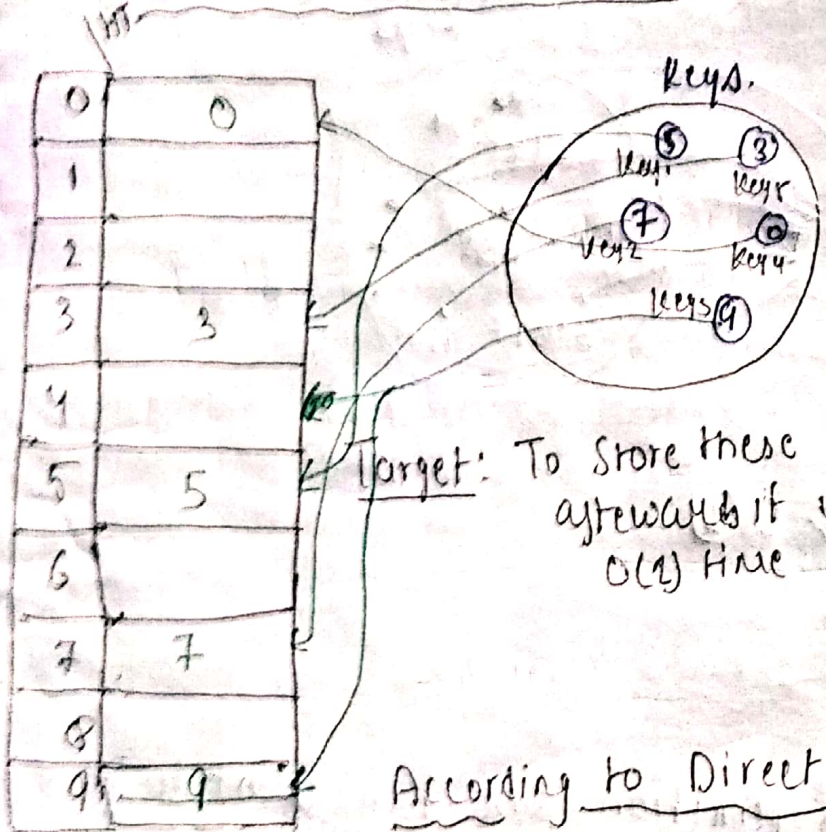


Hashing:

- ① It is one of the searching Technique.
- ② Worst Case Searching Time Complexity - $O(1)$

Direct Address Table:



Target: To Store these keys inside the table. afterwards if we search so it will take $O(1)$ time

According to Direct Address Table:

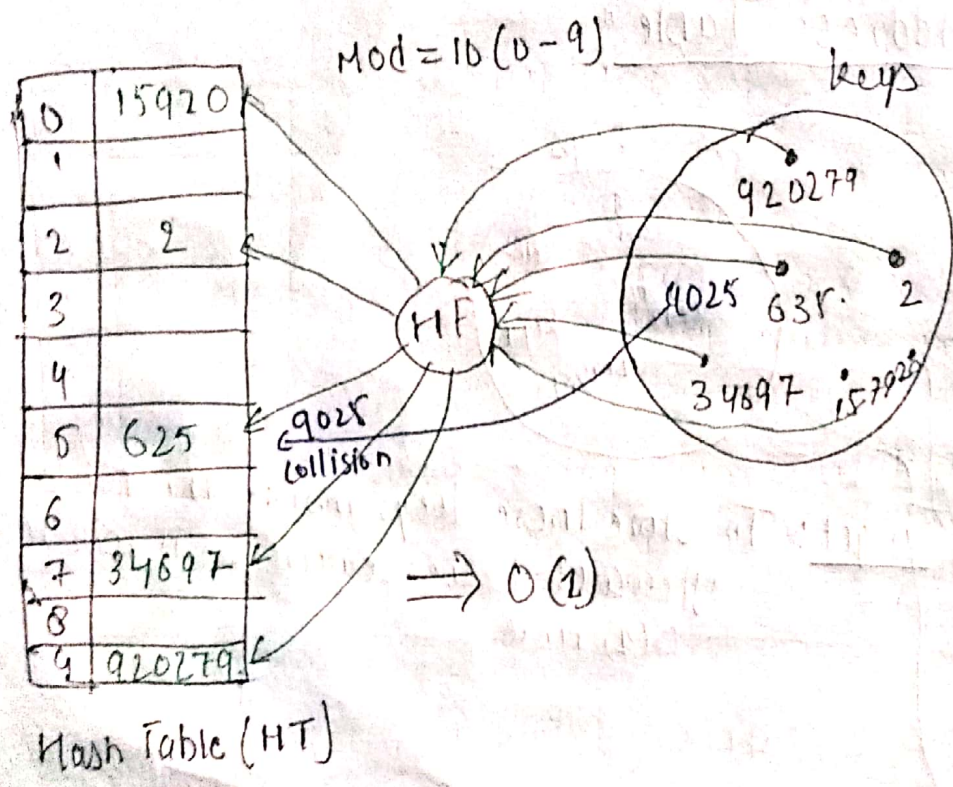
- ① Key Itself is the address without any calculation or without any modification
- ② Searching Time = $O(1)$ [EC]

③ The Drawback of the D.A-T. is even the No. of keys ^{with} is very less but if one of the key is very large 2^{1000} then there is need of Hash table 2^{1000} of size.

④ To Eliminate above drawback. we are going to the hash function.

Hash function:

probe = search = attempt



After Applying hash function. one key point to new hash index called collision.

Disadvantage/Drawback: Collision.

Types of hash function:

① Division Module Method.

Ex ①:
 $M = 1000 (0-999)$
 $key = 789456123$
 $HF(key) = key \text{ Mod } M = 123$

0	
1	
...	
23	789456123
...	
999	

Ex 2: $M = 8$ (2³)

key = 10101010101010

$$\begin{aligned}
 HF(\text{key}) &= \text{key} \bmod M \\
 &= \underbrace{101010101010}_{10 \text{ bit}} \underbrace{100}_{\text{LSB}} \bmod 2^3 \\
 &= 100 \quad (\text{02 0-7})
 \end{aligned}$$

$$\underbrace{11100010010100101010 \dots 1010100}_{10 \text{ bit}} \bmod 2^3$$

= 100 ✓ 10 bit

If bit is 10 or 101 then it always give LSB 3 bit so the major drawback.

Ex 3:

$M = 2$

$$1000110101011101001011100 \bmod 2^k = \underbrace{101100}_{\text{LSB-k bit}}$$

Counter possible if M is power of 2

NOTE: Drawback of Division Module Method

① Don't pick M values exactly powers of 2 because $M = 2^k$. Then the Hash-function of key is = LSB key bit always.

$$HF(\text{key}) = \text{key} \bmod M.$$

② Pick M value which is its prime number. but it cannot be very close to powers of 2.

Ex: 511 Prime No

which is very close to power of 2 = 512

$$\left[\begin{array}{l}
 5 \times 10^3 \\
 701 \\
 \text{Prime}
 \end{array} \right] \text{ It is far from power of 2 }$$

② Mid-Square Method:

- ① First Square
- ② Then take middle.

Best Compare
① 20

$M = 1000 (0-999)$

Key = 84526

$HF(\text{key}) = (84526)^2$

= 7144644676

(Even No so take any so we cannot find the middle so take any two)

but take 3 digit coz max index slot is in 3 digit. So that we can store many digit.

if we take only 6 so slot will be 0-9 so many collision will be there. (Majority address same)

0	
⋮	
469	84526
⋮	
999	

Here every digit will be participated in Division Modulo Method only LSB 3 digit will be participated.

Worst compare previous 2 Method.

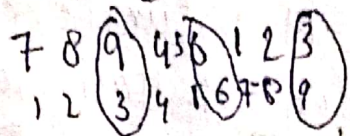
③ Digit Extraction Method:

$M = 1000 (0-999)$

Key: 789456123

$HF(\text{key}) = 963$

3rd 6th 2nd will be extracted



In exam it will be given which will key will be extracted.

963 — 789456123

① Folding Method: Perform Best as compare with

② Fold Boundary Method:

① 2 3

$M = 1000$ (0-999)

Key = 789456123

Take 3 digit from first & last, ^{boundaries.} and the Add.

$HE(\text{key}) = 789$

$\frac{123}{912}$

- If carry come

1412

$\frac{141}{+2}$

143

store in

912 - 789456123

Ex = ~~789~~ Where collision will occur

789683123

$\frac{789}{123}$

912

= 912 - 789683123

same slot different value
(collision)

(ii) Fold Shifting: Here every 3 elements will be fold.

789456123

Start from left side

$\frac{789}{456}$
 $\frac{123}{1388}$

136

+ 8

144

← 789456123

Collision Resolution Techniques

Chaining

open Addressing

Linear Probing

Quadratic Probing

Double Hashing

① Chaining:

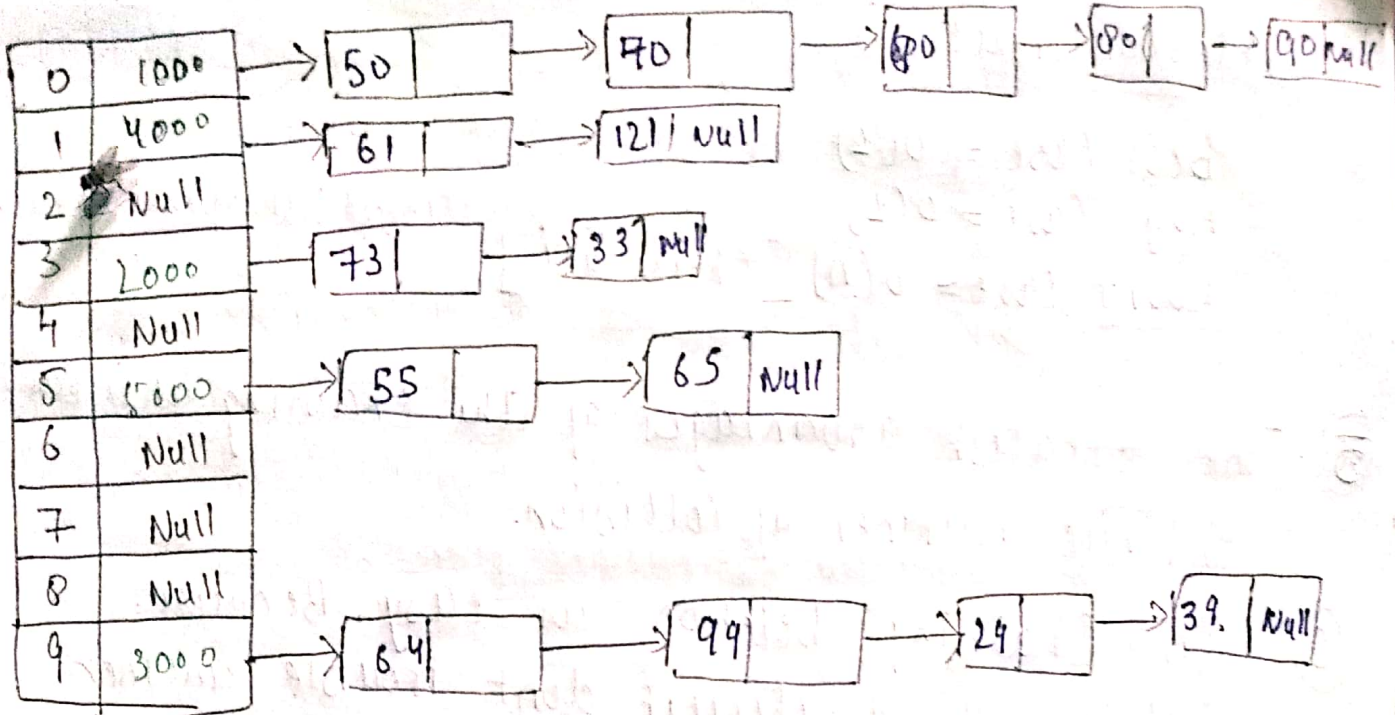
$$M = 10(0-9)$$

$$HF(K) = K \text{ Mod } M$$

Keys = 50 69 73 70 99 40 33 29 61 80 55
65 39 90 121

HCRT = Chaining. (means every key outside)

0	50	70 40 80 90
1	61	121
2		
3	73	33
4		
5	55	65
6		
7		
8		
9	69	99 29 39



Max Chain = 5 Min = Null

Array of pointers to we will take to implement.

In worst case it will give $O(n)$ but it is very in very rare incident because we use best hash method/func so which is not possible. but in very rare chance.

Greatest thing about this: We can store infinite elements.

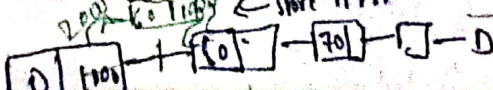
NOTE:

① Lot of space we are wasted. outside in the form of linked list even the space available inside.

② Searching Time = $O(1)$ [BC, Avg. Case]
 = $O(n)$ [most probably all keys will go to same slot. but it very rare] [WC]

③ Insertion Time = $O(1)$ [BC, AC, WC]

Worst Case = $O(1)$ if new key come placed at first no need to insert at last as it is linked list.



④ Deletion Time:

Best Case = $O(1)$

Aug. Case = $O(2)$

Worst case = $O(n)$ [check all] ^{in linked list of that address.}

⑤. The greatest advantages of the chaining is it can handle infinite number of collision.

⑥ In Chaining Deletion is easy. Because deletion of 1 element don't trouble another element.

Open-Addressing:

Linear-probing: (also called closed hashing) *

ex: $M = 10(0-9)$.

$$HF(k) = k \text{ MOD } M$$

Keys = 55, 99, 65, 36, 49, 14, 85, 39, 88.

CRT = L.P.
|||

1st $i=0$
2nd $i=1$
3rd $i=2$
!

$$LP(\text{Key } i) = (HF(\text{Key}) + i) \text{ MOD } M$$

$$\forall i \in \{0, 1, 2, \dots, M-1\}$$

$M = 10(0-9)$

$$LP(55, 0) = (HF(55) + 0) \text{ MOD } M \\ = 5 + 0 = 5$$

$$LP(99, 0) = (9 + 0) = 9$$

$$LP(65, 0) = (5 + 0) = 5 \quad \text{--- collision}$$

$$\cdot (65 + 1) = 5 + 1 = 6$$

VT	
0	49
1	40
2	39
3	88
4	
5	55
6	65
7	36
8	88
9	99

$$LP(36,0) = (36+0) \text{ MOD } 10 = 6+0 = 6 \quad \text{- collision}$$

$$LP(36,1) = (36+1) \text{ MOD } 10 = 6+1 = 7$$

$$LP(49,0) = 9+0 = 9$$

$$LP(49,1) = 9+1 = 10 = \underline{0} \quad \text{- collision}$$

$$LP(40,0) = (40+0) \text{ MOD } 10 = 0+0 = 0$$

$$LP(40,1) = (40+1) \text{ MOD } 10 = 1$$

$$LP(85,0) = 5+0 = 5 \quad \text{collision}$$

$$LP(85,1) = 5+1 = 6 \quad \text{collision}$$

$$LP(85,2) = 5+2 = 7 \quad \text{collision}$$

$$LP(85,3) = 5+3 = 8 = \text{collision}$$

$$LP(39,0) = (39+0) \text{ MOD } 10 = 9+0 = 9$$

$$LP(39,1) = 9+1 = 10 = 0 \quad 3-C.$$

$$LP(39,2) = 9+2 = 11 = 1$$

$$LP(39,3) = 9+3 = 12 = 2$$

$$LP(88,0) = 8+0 = 8$$

$$LP(88,1) = 8+1 = 9$$

$$LP(88,2) = 8+2 = 10 = 0$$

$$LP(88,3) = 8+3 = 11 = 1 \quad 5-C$$

$$LP(88,4) = 8+4 = 12 = 2$$

$$LP(88,5) = 8+5 = 13 = 3$$

ans: $M = 10(0-9)$

$$HF(k) = k \text{ MOD } M$$

$$CRT = LP$$

After inserting 6-keys Table Shown - below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

① Correct Insertion Sequences?

a) 46 42 34 52 23 33

b) 34 42 23 52 33 46

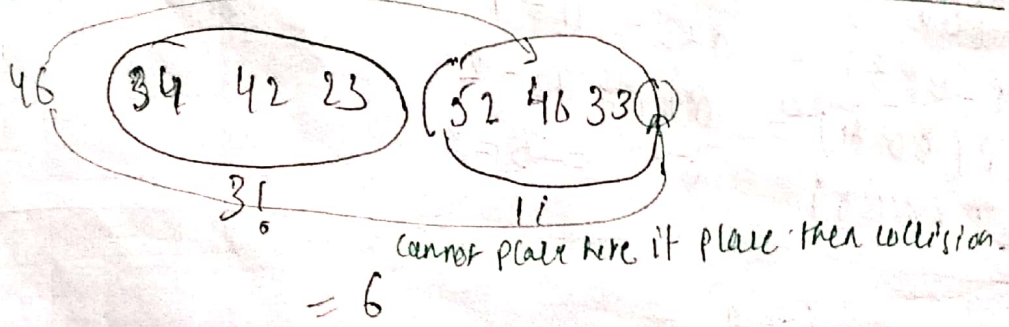
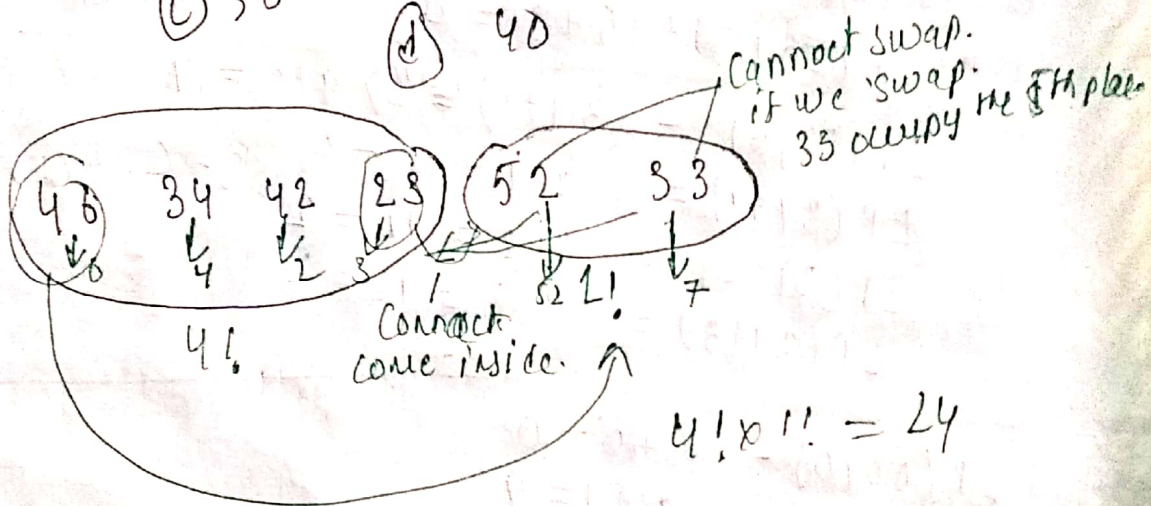
✓ c) (46 34 42 23) 52 33

d) 42 46 33 23 34 5

Insertion Sequence possible?

4, 2
6, 5, 4, 3, 2, 1

- a) 10
- Ⓐ 20
- Ⓑ 30
- Ⓓ 40



Total = $24 + 6 = 30$

Whenever no collision many possibility can be found.

③ Quadratic Probing:

$$M = 10(0-9)$$

$$HF(k) = k \bmod M$$

$$key = 55, 99, 65, 36, 49, 40, 85, 39, 88$$

$$CRT = OP \quad \text{If Given} \quad C_1 = 1, C_2 = 1$$

$$OP(key, i) = (HF(key) + C_1 i + C_2 i^2) \bmod M$$

If C_1 & C_2 are not given

$$\text{then} = (HF(key) + i^2) \bmod M.$$

HT	
0	40
1	44
2	.
3	.
4	.
5	55
6	36
7	65
8	88
9	99

$$OP(55, 0) = 5 + 1 \cdot 0 + 1 \cdot 0^2 = 5$$

$$OP(99, 0) = 9 + 1 \cdot 0 + 1 \cdot 0^2 = 9$$

$$OP(65, 0) = 5 + 1 \cdot 0 + 1 \cdot 0^2 = 5$$

$$65, 1 = 5 + 1 \cdot 1 + 1 \cdot 1^2 = 7 \quad \text{1-C}$$

$$OP(36, 0) = 6 + 1 \cdot 0 + 1 \cdot 0^2 = 6$$

$$OP(49, 0) = 9 + 1 \cdot 0 + 1 \cdot 0^2 = 9$$

$$OP(49, 1) = 9 + 1 \cdot 1 + 1 \cdot 1^2 = 11 = 1 \quad \text{—C}$$

$$OP(40, 0) = 0$$

$$OP(85, 0) = 5 + 1 \cdot 0 + 1 \cdot 0^2 = 5$$

$$= 5 + 1 \cdot 1 + 1 \cdot 1^2 = 7$$

$$5 + 1 \cdot 2 + 1 \cdot 2^2 = 11 = 1$$

$$5 + 1 \cdot 3 + 1 \cdot 3^2 = 7$$

$$5 + 1 \cdot 4 + 1 \cdot 4^2 = 5 \quad \text{—10-C}$$

$$5 + 1 \cdot 5 + 1 \cdot 5^2 = 5$$

$$5 + 1 \cdot 6 + 1 \cdot 6^2 = 7$$

$$5 + 1 \cdot 7 + 1 \cdot 7^2 = 7$$

$$5 + 1 \cdot 8 + 1 \cdot 8^2 = 7$$

$$5 + 1 \cdot 9 + 1 \cdot 9^2 = 5$$

DP(39,0)

39 not assigned.

{10-C

DP(88,0) = 0

Total = 22 collision

③ Double Hashing:

M = 10(0-9)

Hf1(k) = k mod M Hf2(k) = 1 + (key mod (M-2))

keys = 55, 99, 65, 36, 49, 40, 85, 39, 88

CRT = DH.

DH(key, i) = (Hf1(key) + i * Hf2(key))
{forall i in {0, 1, 2, 3, ..., M-1}}

0	40
1	49
2	
3	85
4	
5	55
6	36
7	65
8	88
9	99

DH(55,0) = 5 + 0 * Hf2(55) = 5

DH(99,0) = 9 + 0 * Hf2(99) = 9

DH(65,0) = 5 + 0 * Hf2(65) = 5

DH(65,1) = 5 + 1 * Hf2(65)

65 mod 9 = 2

5 + 1 + 1 = 5 + 2 = 7

DH(36,0) = 6 + 0 = 6

DH(49,0) = 9 + 0 * 2 = 9

= 9 + 1 * (1+1)

= 9 + 2 = 11 = 1

$$DH(85,10) = 5 + 0.6 = 5$$

$$5 + 1.6 = 11 = 1$$

$$5 + 2.6 = 17 = 7 \quad r-3-C$$

$$5 + 3.6 = 3$$

$$39,10 = 9 + 0.8 = 9$$

$$= 9 + 1.8 = 17 = 7$$

$$10-C \quad - \quad 9 + 2.8 = 5 \quad 14-C$$

$$9 + 3.8 = 3$$

$$9 + 4.8 = 4$$

1
:
:

ex 2 $M = 10(0-9)$
 $HF(K) = K \text{ MOD } M$
 Keys = 53, 61, 70, 92, 90, 50
 CRT = L.P.

0	70
1	61
2	92
3	53
4	90
5	50
6	
7	
8	
9	

$$LP(90,10) = 0 + 0 = 0$$

$$0 + 1 = 1$$

$$0 + 2 = 2$$

$$0 + 3 = 3$$

$$0 + 4 = 4$$

$$LP(50,10) = 0 + 0 = 0$$

$$0 + 1 = 1$$

$$0 + 2 = 2$$

$$0 + 3 = 3$$

$$0 + 4 = 4$$

$$0 + 5 = 5$$

Major Drawbacks
 also searching from
 0 instead of 4
 afterwards.
 (Primary clustering
 problem).

① We are probing 1 by 1 slot or 1 slot after another

$$\textcircled{2} \text{ Searching Time} = O(1) [BC] \\ = O(M) [WC]$$

③ Primary Clustering: If two keys contain same starting hash address

then they will follow same path unnecessarily in linear manner. because of this reason average searching time will increase this problem is known as primary clustering.

'Linear probing' is having the drawback of its primary clustering average searching time is equal to (Avg. ^{searching} Access ^{time more} Drawback.

Avg. Searching Time = $1 + 2 + 3 + \dots + n$
1 will take 1 - Unnecessarily path.
2 have to wait for 2.

$$= \frac{M(M+1)}{2} = \frac{M+1}{2} = O(M)$$

for all $\frac{\quad}{M}$

④ Insertion Time = $O(1) [BC]$
= $O(M) [WC]$
= $O(M) [AC]$

⑤ Deletion Time = $O(1) [BC]$
= $O(M) [WC]$
= $O(M) [AC]$

⑥ Deletion Difficult because 1-element. Deletion trouble other elements But we can manage with special symbol \$

If more \$ then re-hash again.

Ex ②: $M = 10(0-9)$
 $HF(K) = K \text{ MOD } M$

Keys = 53, 61, 70, 92, 90, 50.

0	70
1	61
2	92
3	53
4	
5	
6	90
7	
8	
9	

OP(90,0) = $0 + 1 \cdot 0 + 1 \cdot 0^2 = 0$
 $= 0 + 1 \cdot 1 + 1 \cdot 1^2 = 2$
 $= 0 + 1 \cdot 2 + 1 \cdot 2^2 = 6$
 3 Comparison

OP(50,0) = $0 \cdot 1 \cdot 0 + 1 \cdot 0^2 = 0$
 $= 0 + 1 \cdot 1 + 1 \cdot 1^2 = 2$
 $= 0 + 1 \cdot 2 + 1 \cdot 2^2 = 6$
 $= 0 + 1 \cdot 3 + 1 \cdot 3^2 = 12 = 2$
 !

It will take more than 3rd Comparison! Min=3 So redundancy there. unnecessarily comp. (Secondary Clustering)

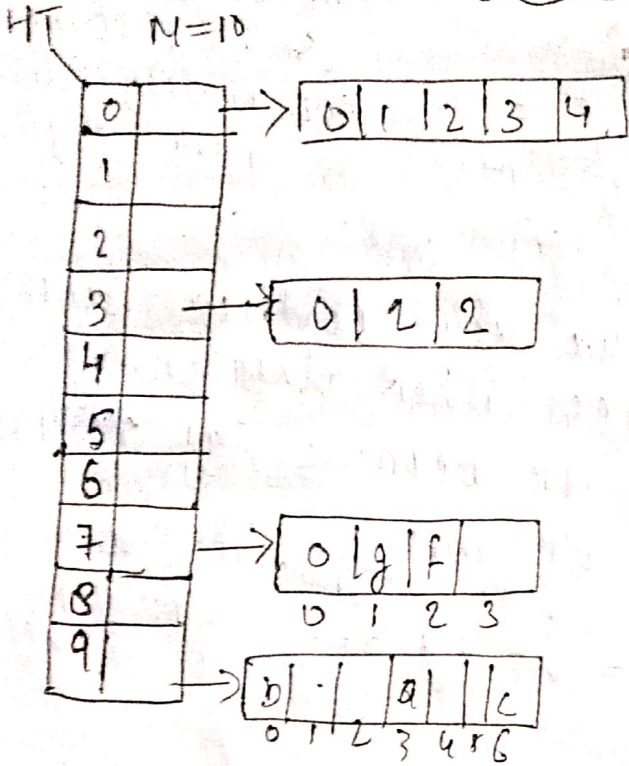
① Secondary Clustering: If two keys contain same starting has Address they both will follow same path. unnecessarily in Quadratic manner. because of this reason. Average searching time will increase. this problem is known as Secondary Clustering.

② Quadratic Probing is having the draw-back of Secondary Clustering.

② Searching Time = $O(1)$ [BC]
 $O(N)$ [WC]

16-1 Sept-2016

Using perfect Hashing you will get worst case time-complexity. $O(N)$.
Perfect Hashing



$h_0(d) \rightarrow 0$
 $h_0(e) \rightarrow 0$

$h_0(d) = 4, h_1(e) = 0$

$h_0(a) \rightarrow 9$
 $h_0(b) \rightarrow 9$
 $h_0(c) \rightarrow 9$

$h_a(a) = 3, h_a(b) = 0$
 $h_a(c) = 6$

$h_0(f) \rightarrow 7$
 $h_0(g) \rightarrow 7$

$h_7(f) = 2, h_7(g) = 1$

Time complexity $\div O(N)[\epsilon C]$

for every slot more than one hash function & hash table are there.

Universal hash function is independent from key.
 called Universal hash function

Uniform hash function. gives priority to every slot. equal & can go to any slot, equal probability

$M = \text{Number of slots}$

$N = \text{Number of keys}$

$M = \text{slots} = N - \text{keys}$

1. Slot = $\frac{N}{M}$ keys/slot. (In 1 slot how many)
 or
 Average or $\frac{\text{Number of elements}}{\text{per slot}}$

Load factor (α)

Load factor (α) for Chaining. $0 \leq \alpha \leq \infty$
 Open Addressing

$$0 \leq \alpha \leq 1$$

① The expected number of probes in an unsuccessful search in open Addressing Technique = $\frac{1}{1-\alpha}$

② For successful search = $\frac{1}{\alpha} \ln \frac{1}{1-\alpha}$

Programming

① Scope of a Variable

(i) Static Scoping.

(ii) Dynamic Scoping.

Ex ①

int a = 5;

Main()

```
{
  int a = 10;
  B();
}
```

B()

```
{
  int a = 20;
  C();
}
```

C()

```
{
  int a = 30;
  D();
}
```

D()

```
{
  int a = 40;
  P(a);
}
```

allocate mem. at compile time (static area)

allocate at run time (stack area)

Local variable memory will create at run-time.
 • If there is one function (C) No one can call. In that function so many declaration. So what the need of creating memory for them. So that's the reason. Local variable allocate at run-time.

Stack area is a small area. If you didn't assigned any value. By default it will zero. Instead of garbage value. (Small area you can clean, but delhi you can not clean).

Compiler \Rightarrow HLL to LL lang.
 (Translator / Converter).

Processor only understand Low-Level-lang.

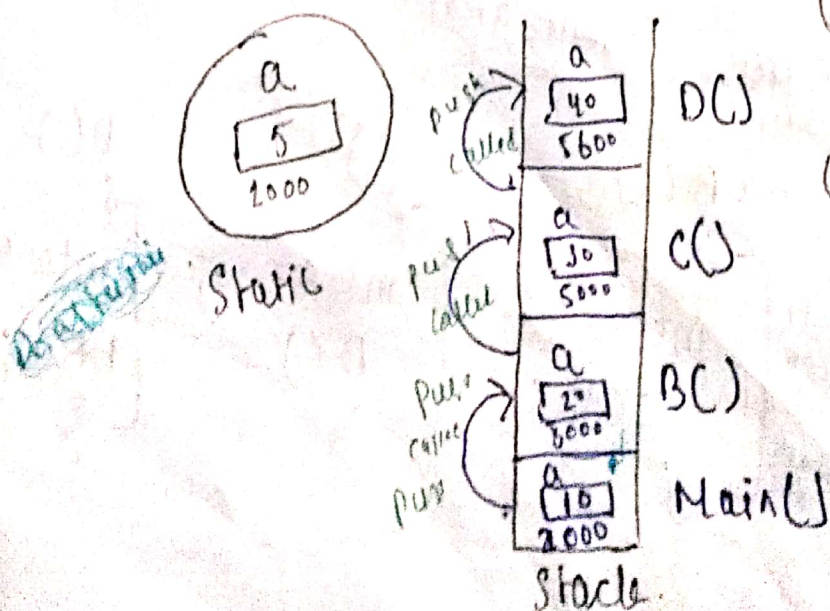
$a + b \rightarrow \text{ADD } R_0, R_1$
 HL. LL
 (User Convenience) (CPU Convenience)

① First he check or verify it is my program or not.
 It convert HL to LL after checking syntactic & semantic.
 Logical error: Semantic error
 Physical error: Syntax "

After compilation over:

Rule:

- ① Within the function you cannot declare. dual variable with same name
- ② w/o without declaring a variable, you cannot use it.



Scope problem comes if there is no declaration of local variable & you are calling a function. Compiler confused which variable values should a print.

How to resolve the scope problem.

You say compiler to resolve the problem there is two ways

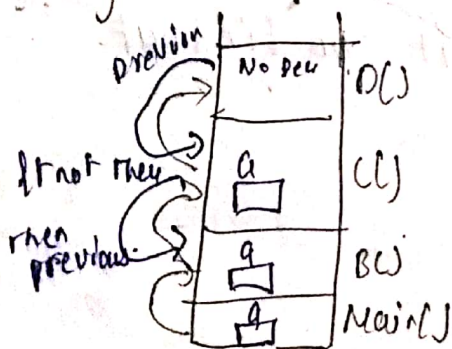
(i) Static Scoping: During compile time. Compiler will resolve static scoping using global variable static values

(ii) Dynamic Scoping: Processor will resolve dynamic scoping taking previous value as a (handling very difficult)

Every programming follow static scoping

```
D()
{
  int a = 10 - No declaration.
}
```

Pf(a) So static scoping will take static value
Dynamic will take previous value.



In previous first example (2) In D function No Declaration here. This function is in the top of stack because of no declaration of a. Compiler confused which value of 'a' should print. coz there are many value of a. This problem is called scoping problem.

To handle the problem two ways are there

- ① Static Scoping
- ② Dynamic Scoping

Compiler solve very easily because very less work. but in Dynamic type checking. it means you are checking at runtime. so very work you have to do. checking previous one. if a is not then. then previous. and up to so on. So to handle Dynamic Scoping it is very difficult so every ~~pr~~ ~~g~~ lang follows. Static type checking. It will take less time. coz it will scan one type. at compilation time.

Dynamic Type Checking:

- Type Checking done at run-time.
- More time.
- All type related error it cannot find out.

Static Scoping Type Checking:

- Type checking done at compile time.
- It will take less time.
- All type of related error it can find out.

for ex: a=10 b=20

if (a > b)

{

}

else

{

- (Type error here)

}

}

if condition true

it will execute only

if part it never know the problem or error there in else part.

But in static before runtime

it will scan the program

at 1 time. if error it will

give error msg.

Memory allocation should be Dynamic only.

It cannot be static. Compiler cannot estimate how much function will take how much space coz of recursion. (Drawback: It don't support recursion)

Recursive Program should be Dynamic due to some function & variable. calculate again & again.

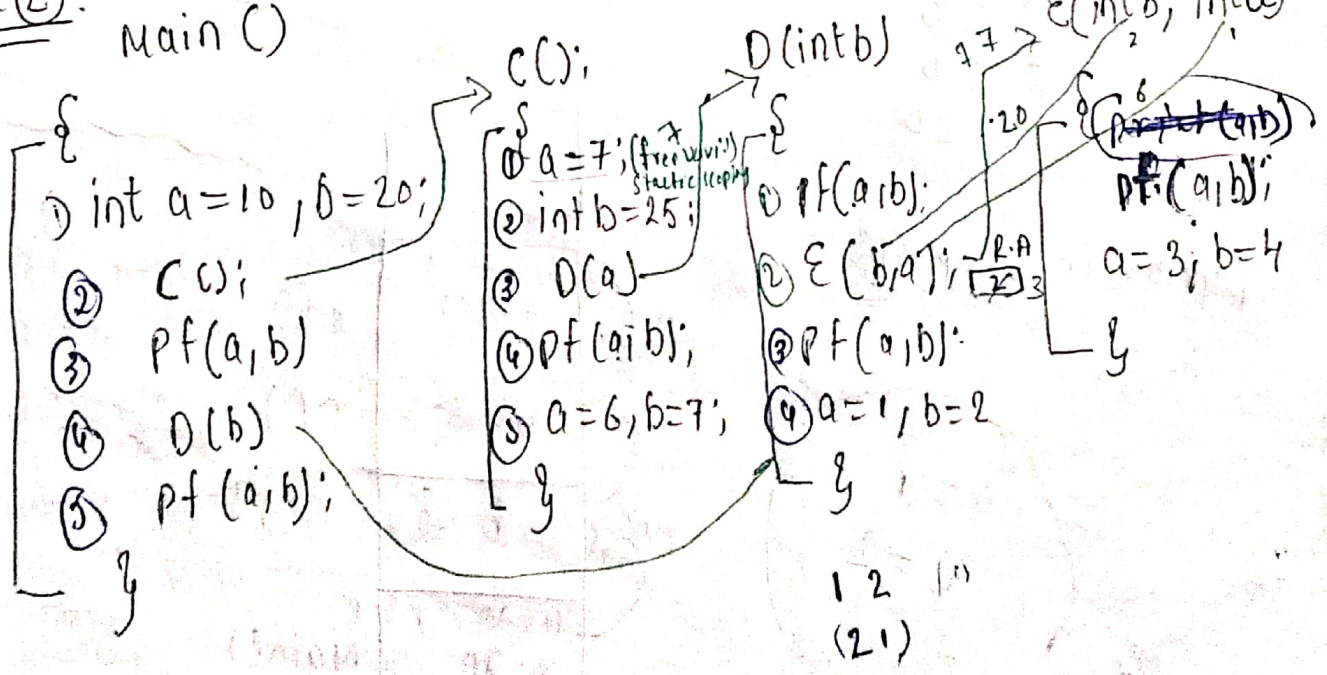
```

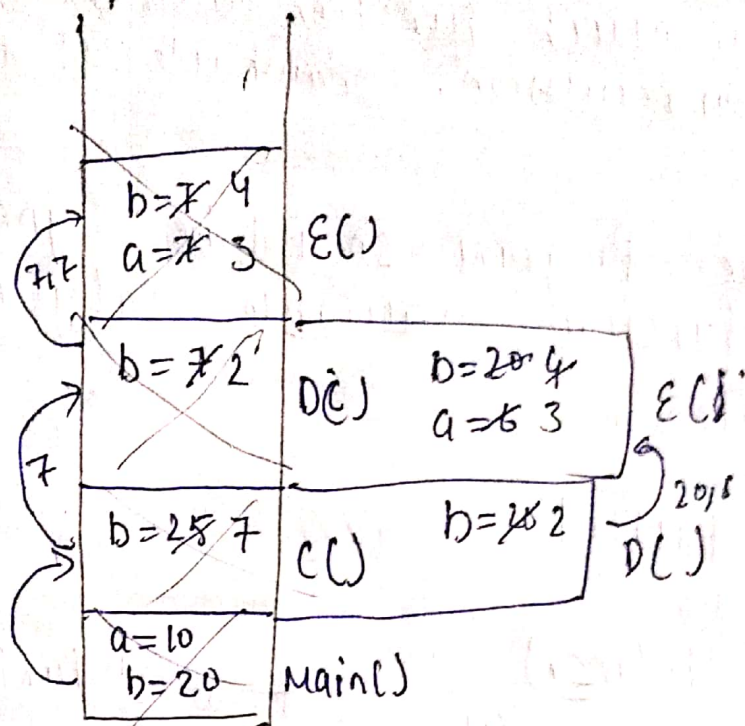
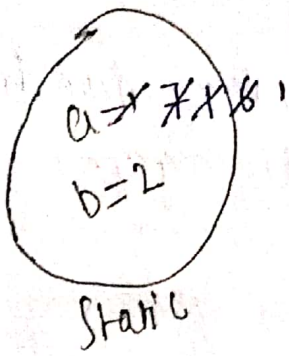
Ex: fact(n)
{
  if (n <= 1)
    return (1)
  else
    return (fact(n-1) * n)
}
  
```

It is 2nd function calls. So it need 2nd stack space. But compiler will check at once & think that it is one line code it will think it will get only 2 stack space.

Static Scoping:

Ex(2): int a=1, b=2
Main ()

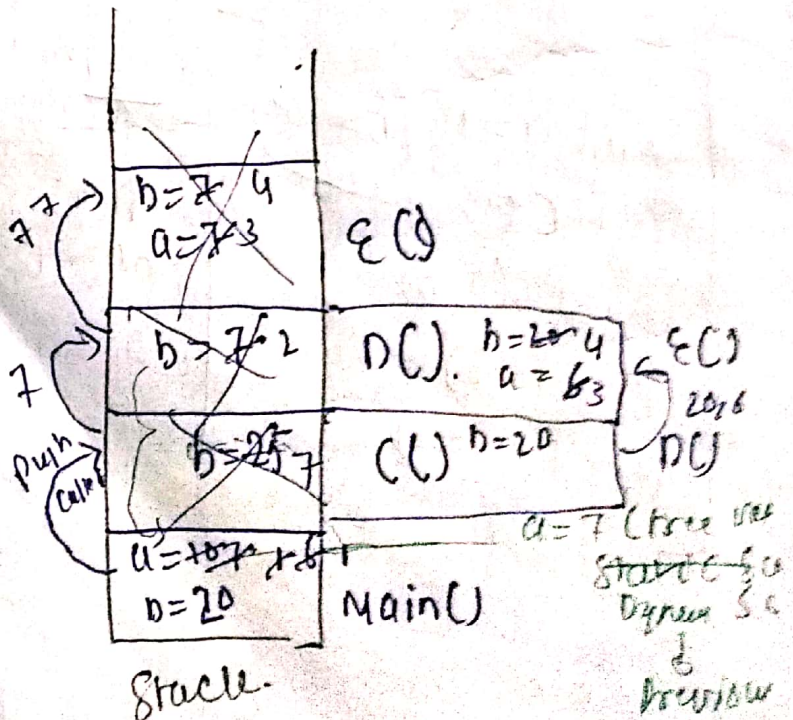
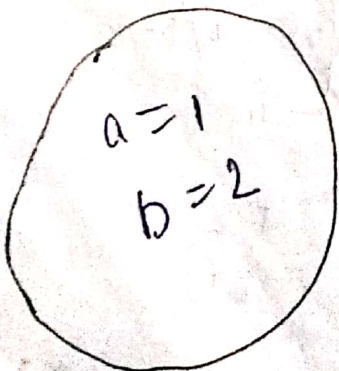




O/p: Static Scoping:

- 7, 7
- 7, 7
- 7, 7
- 1 25
- 10 20
- 6 20
- 6 20
- 6 20
- 10 20

Dynamic Scoping:



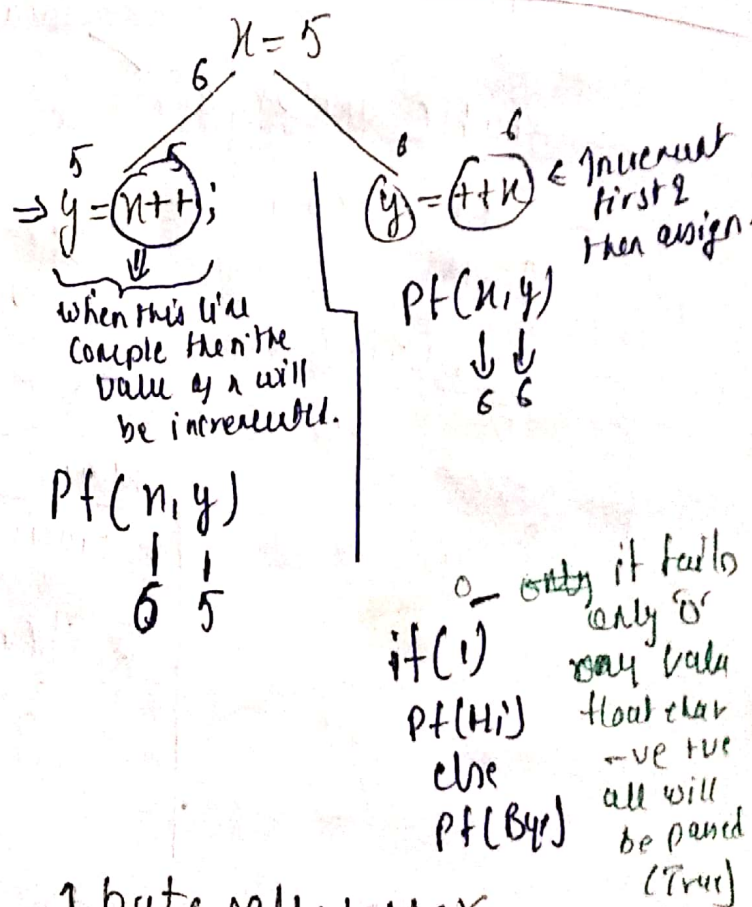
Dynamic Scoping:

O/P:
 7, 7
 7, 7
 7, 7
 10, 25
 6, 20
 6, 20
 6, 20
 6, 20
 1, 20

Static Variables:

```

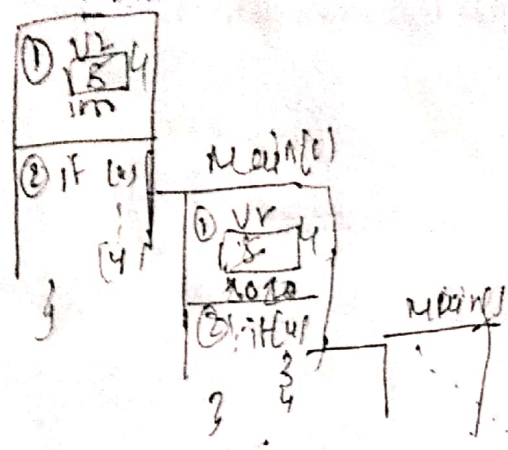
Main()
{
  static int var = 5;
  if (--var)
  {
    main();
    Pf(var);
  }
}
  
```



A thing which can be stored in -2 byte called char
 - 2 is integer.

In Comp. No char no int; float, everything will be based on space (size).
 char we can store it in integer but integer cannot be stored in char. Similarly, float.

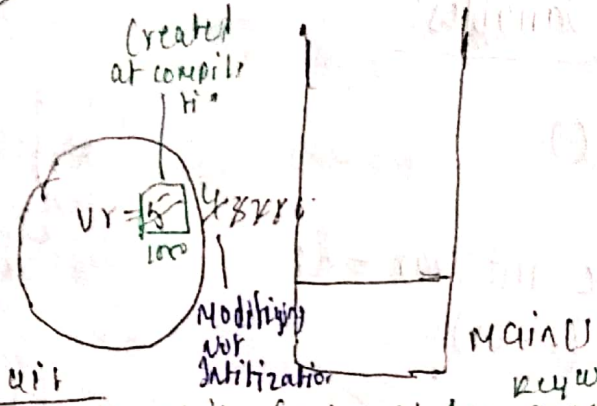
Ex-1 If static Not there.



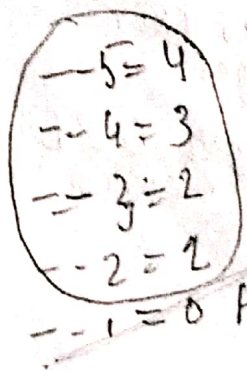
With static.

```
main()
{
1. static int vr=5;
2. if(--vr)
{
3. main()
4. printf
}
}
```

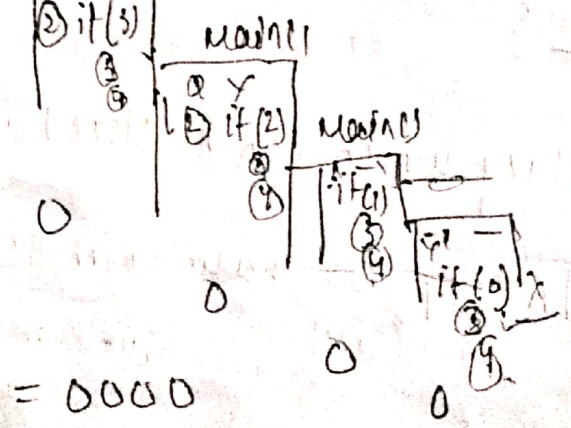
Stack overflow.
local static variable so created at compile time.



While seeing static keyword processor skip this line. it is not his work. it will done with compiler.



from 4 zero will print.



NOTE:

Stack For static variable Memory will allocate in static area. of main. memory.

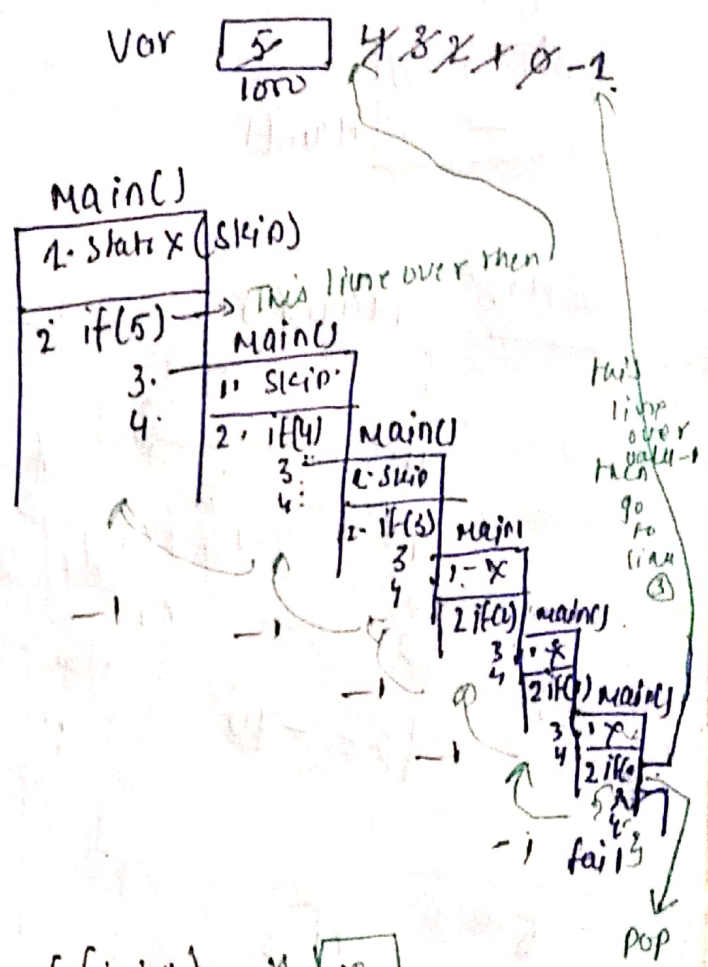
- ① For static variable memory will allocate only once.
- ② Static variable will be initialized only once.
- ③ By default static variable will be initialized to '0'

Ex 2

```

main()
{
  1. static int var = 5;
  2. if (var--);
  {
    3. main();
    4. pf(var);
  }
}

```



O/P: -1 -1 -1 -1 -1

Ex 3

```

Main()
{
  int n = 10, y = 20, i = 1;
  for (i = 1; i <= 2; i++)
  {
    y += f(n) + g(n);
    pf(y);
  }
}

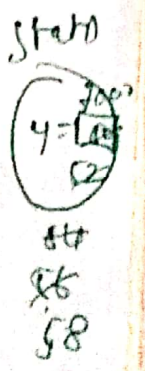
```

```

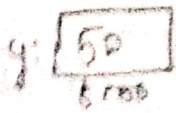
f(int n)
{
  int y;
  y = g(n);
  return (n + y);
}

g(int n)
{
  static int y = 50;
  y += 2;
  return (y);
}

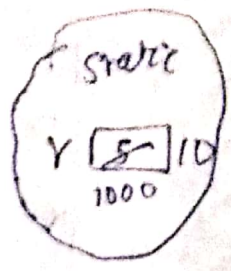
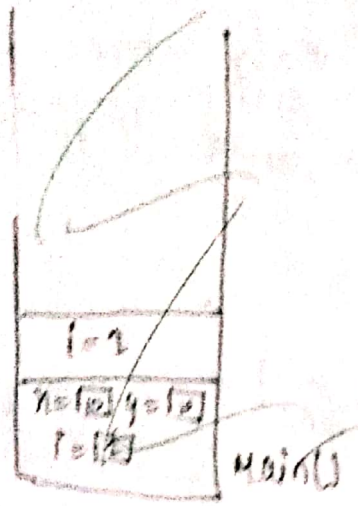
```



int a = 20; ~~initialization~~ (But initialization not done)
 a = a + 5; ~~Modification~~ (A static variable Modification can be done)



O/P: $\frac{156}{300}$

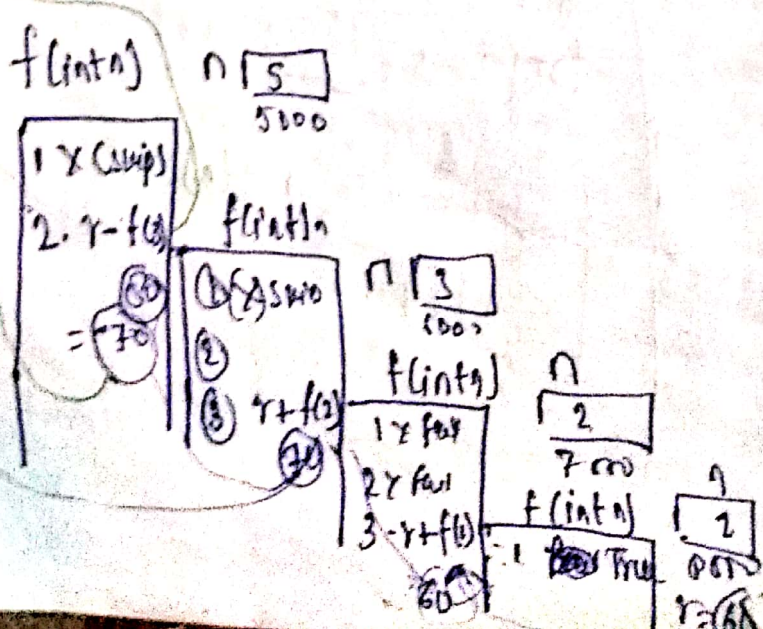
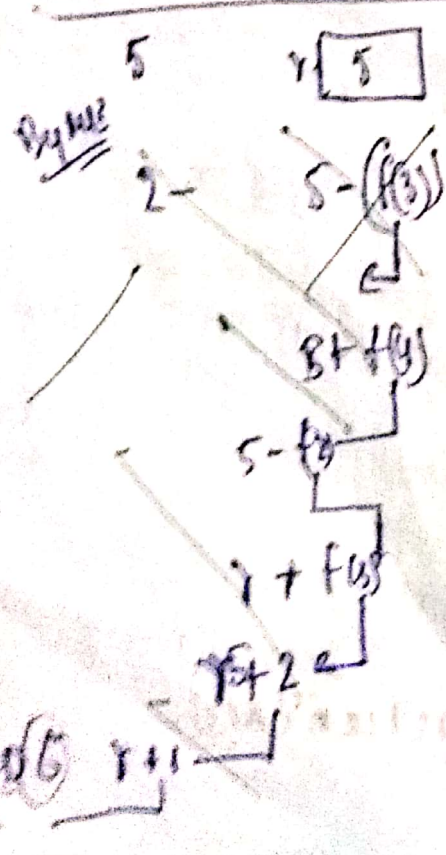


Ex 1: main()
 {
 int n = 5;
 pt ("%d", f(n));
 }
 O/P = -70

f(int n) n:

5
5000

 {
 static int r = 5;
 ① if (n <= 1)
 {
 r = 10;
 return (r + 50);
 }
 ② if (n > 3)
 return (r - f(n-2));
 else
 ③ return (r + f(n-1));
 }



11/9/2019

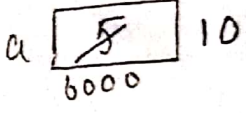
Extern Variable:

```

Exp 1) int a = 5 - Global Var.
      Main()
  {
    extern int a
    Pf(a);
    a = 10;
    Pf(a);
  }

```

while using this keyword. Memory never created at runtime. It will use the memory of global variable. If there is variable which is copy of that (local variable but taking mem. of global)



O/p: 5
10

Exp 2)

```

Main()
{
  extern int a
  Pf(a)
  a = 10;
  Pf(a);
}

```

extern int a → No memory created at runtime coz of extern keyword.
 Pf(a) → No global variable there. No variable there with name a. So error.

O/p: Error Msg. (Undefined)

Exp 3)

```

Main()
{
  extern int a;
  Pf(hi);
}

```

No relation. No need to use the value of a. So print hi. *So don't check at global.*

O/p: Hi

Exp 4)

```

extern int a;
Main()
{
  extern int a;
  Pf(a);
}

```

If you don't initialize memory not created.

O/p: error undefined symbol.

Ex 10

```
extern int a = 10
main()
{
  extern int a;
  pf(a)
  a = 5
  pf(a)
}
```

O/P: 10
5

If you initialize the value then extern never work. So memory will be created. coz extern variable Global.

local variables

```

int a
auto int a
register int a
} All are same
memory will allocate
at register & program will be faster.
```

```
main()
{
  int a = 5;
  int a = 10
  pf(a)
}
```

Multiple declaration error. 2 local with same name.

Within its function with same name. you can create multiple variable. by using the keyword. extern otherwise not possible.

```
int a = 5
main()
{
  extern int a;
  extern int a;
  extern int a;
  extern int a;
  pf(a)
}
```

All are using the Global memory.

Ex 6: A program which contain all the keyword. what we learn before.

```

int a=1, b=2;
Main()

```

```

{
  auto int a=10;
  Pf(a,b);
  C();
  Pf(a,b);
  a=3, b=4;
  D(b);
  Pf(a,b);
}

```

```

C()
{
  static int a=7;
  Pf(a,b);
  a=9, b=10;
  D(a);
  a=21, b=22;
}

```

```

D(int b)
{
  extern int a;
  Pf(a,b);
  a=25, b=27;
  C();
  a=30, b=31;
}

```

```

E()
{
  register int a=29;
  Pf(a,b);
  a=30, b=39;
}

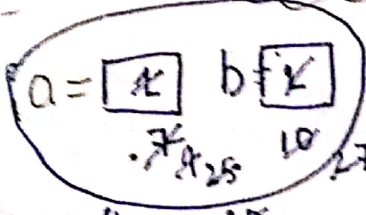
```

Static Scoping

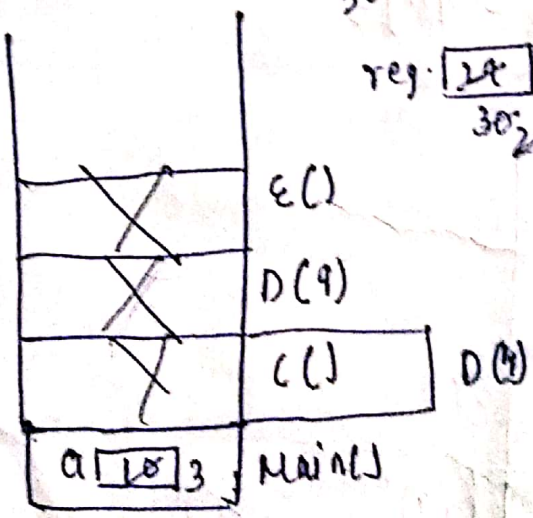
By all:

Dynamic Scoping

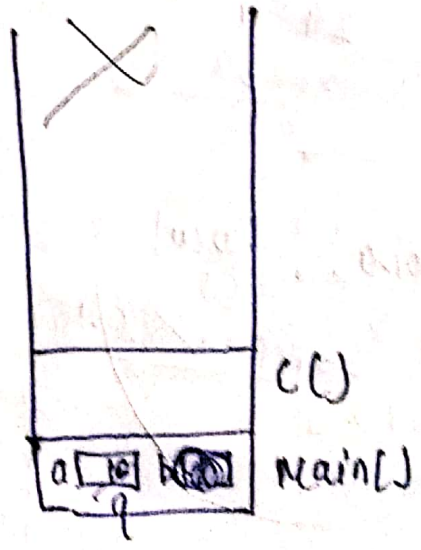
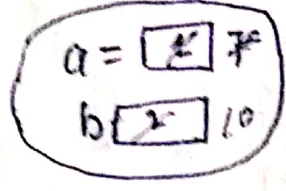
Sol:



Static: 30, 25, 30, 27, 31, 25, 30, 27, 31, 27, 31, 39, 31

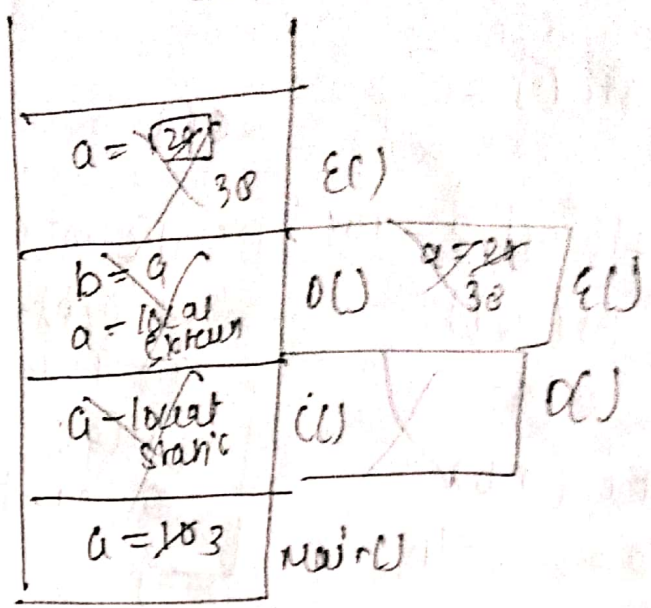
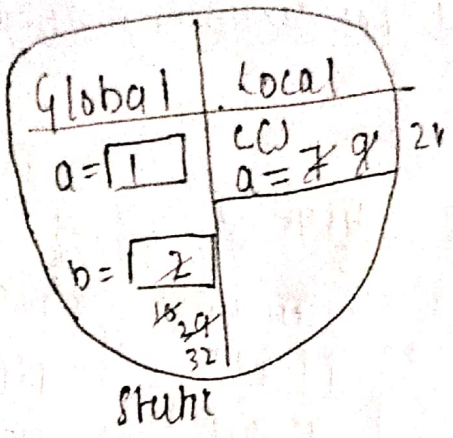


o/p: 10 2
7 2
9 10
29 27
20 22
21 4
29 27
3 31



o/p: 10 2
7 2

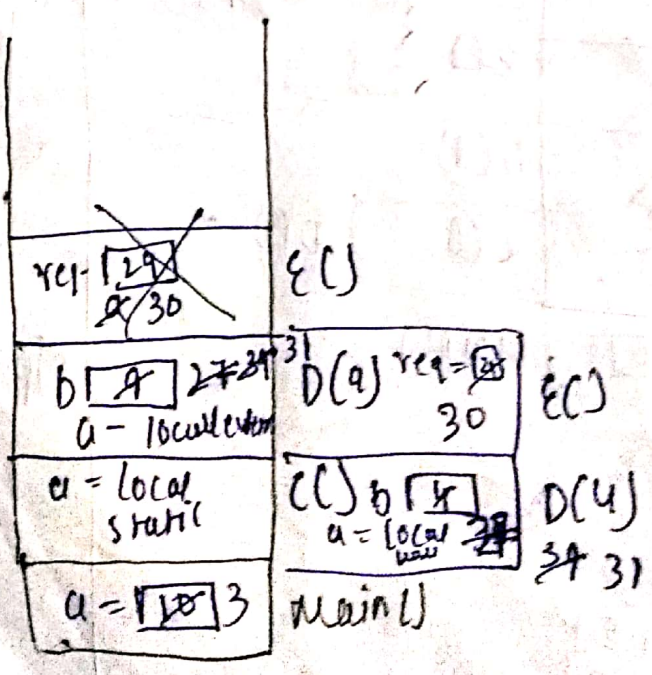
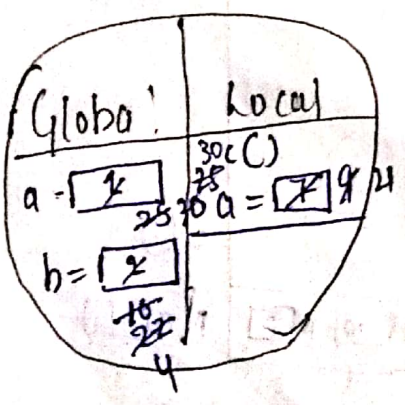
By Site



Stack

O/P: 10 2
7 2
1 9
29 10
10 22
30 4
29 4
3 39

Dynamic Scoping:

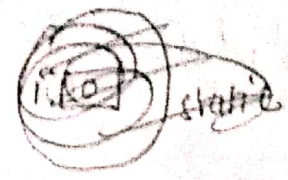


O/P: 10 2
7 2
1 9
29 27
10 22
30 4
29 27
3 4

ex 7:

```

int i;
Main()
{
    int i;
    for (i=1; i <= 25; i++)
    {
        switch (i)
        {
            case 1: i += 3;
            case 2: i += 4;
            case 3: i += 5;
            default: i += 3;
        }
        printf("%d\n", i);
    }
}
    
```



o/p: 16
20
24
28
20
24
28
29

break - No use of break after every case break required

NOTE:

- In Switch stat statement after case. break is needed if not it will executed remaining cases also.
- After default no need of break anyhow it is a last statement.
- In switch statement default can be placed anywhere. but it will execute when no match is there.

& / && Boolean and and operator
^ bitwise

Qp: Main()

```

int i = -2, j = -2, k = -1, l = 2, m;
M = ( (i++) && j++ ) && (k++) || (l++);
    
```

i = -1
j = -1
k = -1
l = 2
m = 1

Pf (i, j, k, l, m);
 ↓ ↓ ↓ ↓ ↓
 0 0 0 2 1
 ↳ whoever came first who will be done first
 () = left → right
 = right → left.
 ↳ for anything = 2 (so need to require)

ex 7:

```

int i;
Main()
{
  int i;
  for (i=1; i <= 25; i++)
  {
    switch (i)
    {
      case 1: i += 3;
      case 2: i += 4;
      case 3: i += 5;
      default: i += 3;
    }
    printf("%d\n", i);
  }
}
  
```



i: 0 24 8
 o/p: 16
 20
 24
 28
 20
 24
 28
 29

break - No use of break after every case break required

NOTE:

- In switch stat statement after case. break is needed if not it will executed remaining cases also.
- After default no need of break anyhow it is a last statement.
- In switch statement default can be placed anywhere. But it will execute when no match is there.

& / && Boolean and and operator
 ~ bitwise AND operator

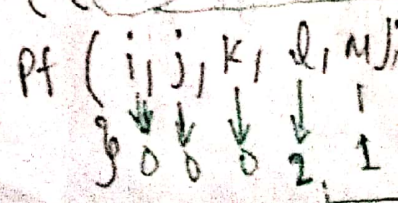
Ques: Main()

```

int i = -2, j = -2, k = -2, l = 2, m;
M = ( (i++) && j++ && (k++) ) || (l++);
  
```

i = -1
 j = -1
 k = -1
 l = 2
 m =

0 && 1 = 0
 1 && 1 = 1
 0 && 0 = 0
 1 && 0 = 0



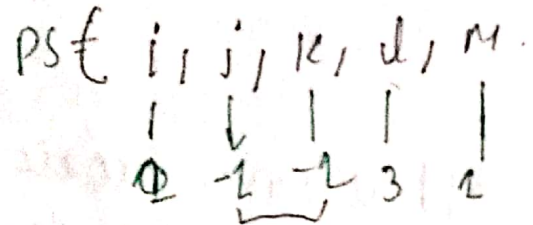
-> Whoever came first who will be done first
 () = left -> right
 == right -> left.
 for anything = 2 (so need to require to execute left)

1 or anything = 1 } called short circuit
 0 & anything = 0

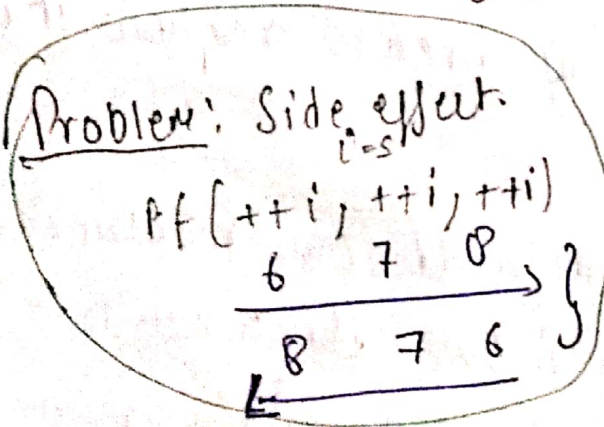
In previous we get ans 2 so we need to execute further. so (1 || i++) never execute. coz before its execution we get answer coz 2 or anything = 1.

If $i=0, j=-2, k=-2, d=2, m$

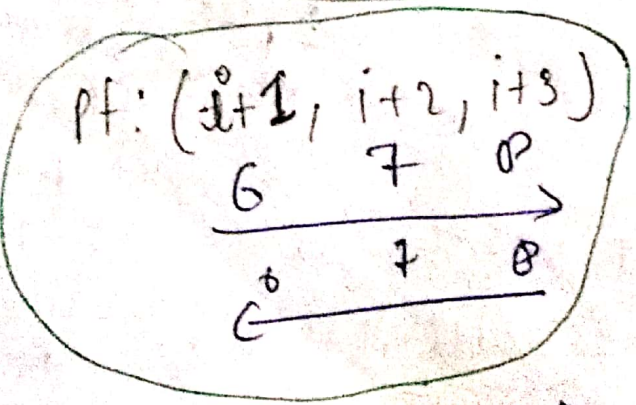
$$m = \underbrace{((i++) \&\& j++ \&\& k++)} || \underbrace{(1++)}_{2}$$



Never execute
 0 & anything = 0 (so no need to execute)



Two Different answer.



no side effect.
 Do left to right.

Every program will be side effect free program.

Always do from left to right.

2018

Pointers

Ex 1

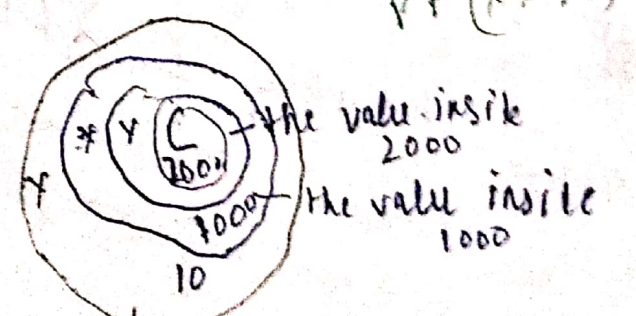
- ① $\text{int } a = 10$
 Declaration OS \Rightarrow $\text{a} \begin{cases} 10 \\ 1000 \end{cases}$
 Def OS \Rightarrow $\text{b} \begin{cases} 1000 \\ 1000 \end{cases}$
 Inside b - some address 8B
- ② $\text{int } *b = \&a$
 So OS will assign the mem. whatever the size of the address
- ③ $\text{Pf}(a) \Rightarrow 10$
- $\text{Pf}(\&b) \Rightarrow 1000$
 user \Rightarrow value of b
- $\text{Pf}(\&(*b)) \Rightarrow 10$

Wrong Declarations

$\text{int } *a = \text{value}$
 $\text{int } a = \text{address}$

Ex 2

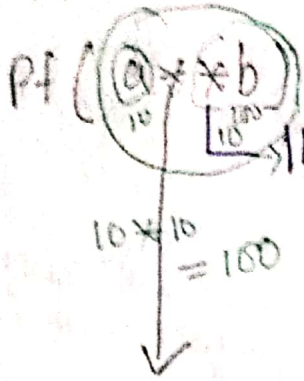
- ① $\text{int } a = 10$
 $\text{a} \begin{cases} 10 \\ 1000 \end{cases}$
- ② $\text{int } *b = \&a$
 read \Rightarrow b is a pointer of type integer (or b is a pointer which contains address of integer)
 $\text{b} \begin{cases} 1000 \\ 200 \end{cases}$
- ③ $\text{int } **c = \&b$
 c is a double pointer of type integer
 $\text{c} \begin{cases} 2000 \\ 3000 \end{cases}$
- $\text{Pf}(a) \Rightarrow 10$
- $\text{Pf}(*b) \Rightarrow 10$
- $\text{Pf}(**c) \Rightarrow 10$
 you will get the value of that address
 you will get " " " " " "
- $\text{Pf}(\&(*c))$ ~~is not allowed~~



More than 3 star or 3 pointer not allowed
 (02 you declare only 2 pointer (2 star)
 So how can we use 3 star. (only 2 or less than 2)

Ex 10

```
int a = 10
int *b = &a
```



It is not multiplication. We will take it as a pointer. coz we declare before `b` as pointer, even processor also confused what we will tell by pointer or multiplication so he will check the declaration. if declare them as then it will take as pointer

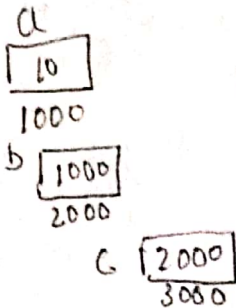
This stage we will take as multiplication coz

we didn't declare the double pointer. so processor first check the declaration. Double pointer is not there so it will take as multiplication.

every pointer = 8 Byte
 size of integer = 2 Byte
 " " char = 1 Byte
 " " float = 4 "

Ex 10

```
int a = 10
int *b = &a
int **c = &b
```



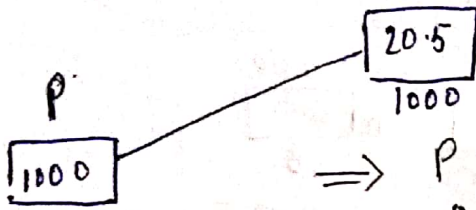
Size of `c` = 8 Byte

Size of `*c` = 8 B 1000 = Address

Size of `**c` = 2 B 10 = value

char.
 => 8 Byte
 => 8 Bytes
 => 2 Byte

P is a pointer means it contain address.
 But which pointer.



⇒ P is a pointer pointing to float.
 or P is a float pointer.
 $Size(P) = 8B$
 $Size(*P) = 4B$

Here we know P is a pointer which contain address.

So how we decide of pointer pointing integer then P is a integer pointer like this



⇒ P is a pointer pointing to Integer
 or P is a Integer pointer

Size of a Pointer(P) = 8B
 Size(*P) = 2B

Ex 6

int i=5, j=7;

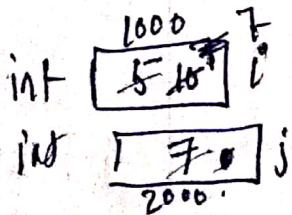
main()

```
{
  f(&i, &j);
  P+(i, j);
}
```

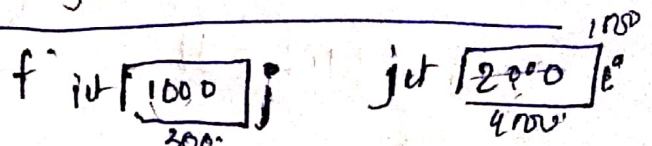
f(int *j, int *i)

```
{
  i = j;
  *i = *j + 5;
  *j = *i - 3;
  j = i;
  *i = *j;
}
```

Sol key we:



o/p = ~~10 7~~
~~7 7~~

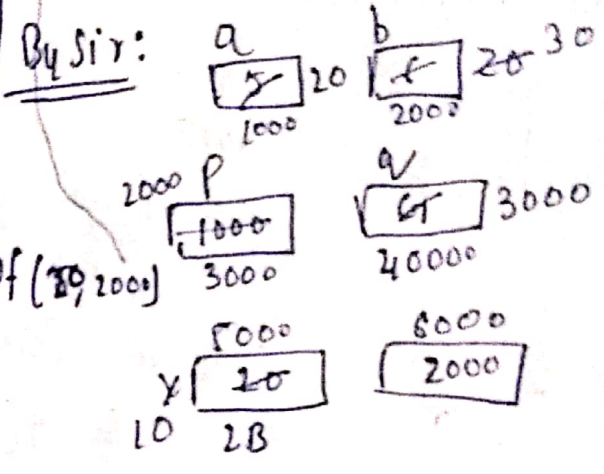
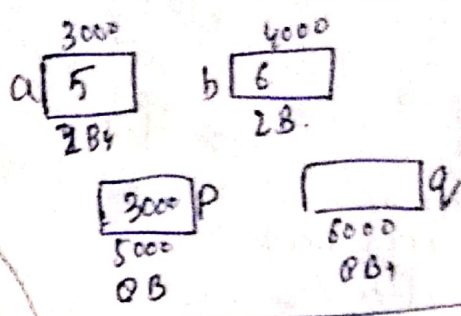
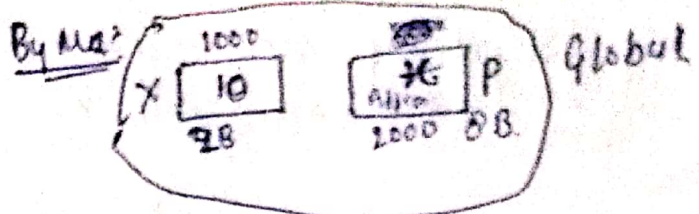


*i = 10
 *j = 5
 *i = *j

```

ex:
f(int x, int *p)
{
    *p = x;
    x = 10;
}
main()
{
    int a = 5, b = 6;
    int *p = &a, *q = &b;
    *p = 20;
    f(a, &b);
    *q = &b;
    *p = 30;
    pf(0, 10);
}

```



O/P: 20, 30

```

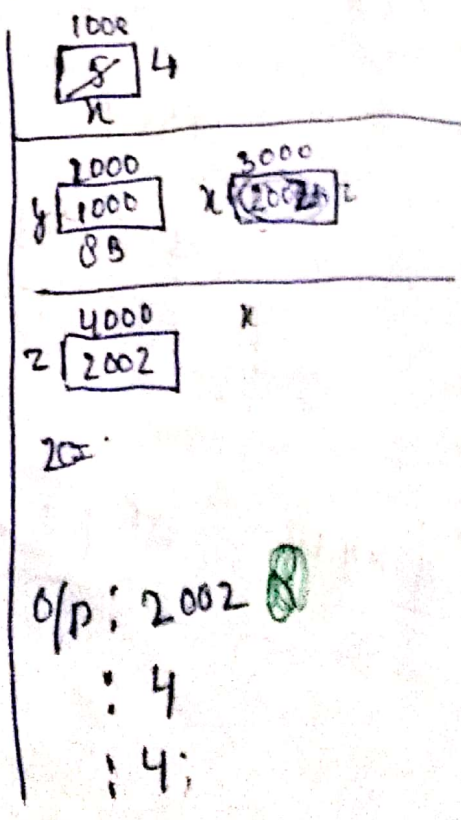
ex:
int n;
void O(int z)
{
    z += n;
    pf(z);
}
void P(int *y)
{
    int n = *y + z;
    O(n);
    *y = n - 1;
    pf(n);
}

```

```

Main()
{
    n = 5;
    P(&n);
    pf(n);
}

```



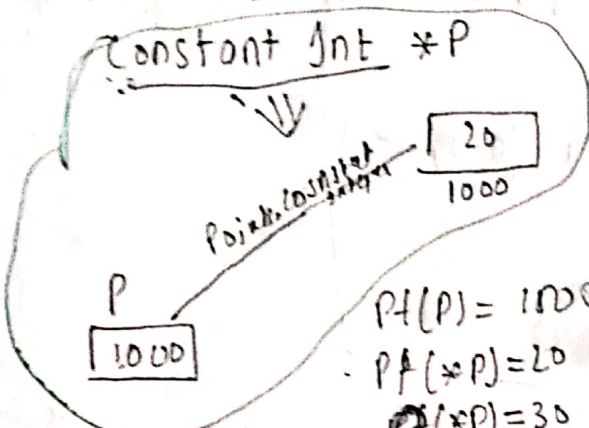
int P
 8 1000
 6 1000

4 1000 2000 x 7 3000
 2 7

O/P = 12, 7, 6

int *constant P

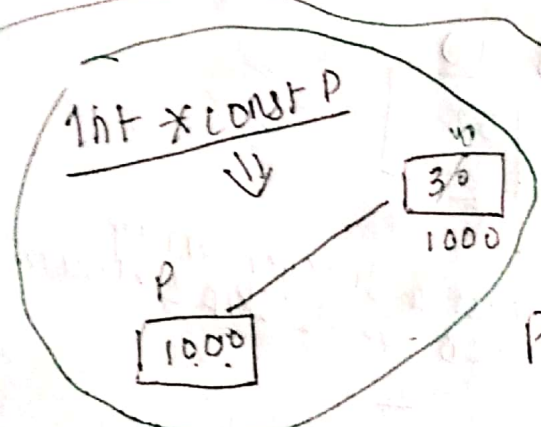
constant int *P



PF(P) = 1000
 PF(*P) = 20
 PF(*P) = 30

error coz we cannot change coz of constant. But here you can change the value of P = 2000

int *const P



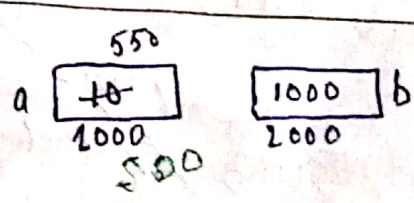
Print P = 1000
 PF(*P) = 30
 *P = 40
 P = 2000

P = 2000 (X Error. Because here P is constant you cannot change it)

Ex 100

Main

```
int a = 10;
int *b = &a;
scanf("%d", b); // 500
printf("%d", a+50); // => 550
```



O/P : TIP 550

Whatever the spp you will take

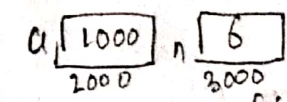
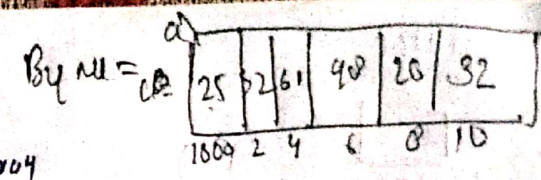
Sept. 12

Main C)

```

{
  int a[] = {25, 32, 61, 98, 26, 32};
  Pf("%d", f(a, 6));
}
  
```

Here a+1 No possible for array



$25/100 = 25 + f(100, 5)$

In array pf(a) = 1080
 a = 1000
 a+1 = 1002

variable = inside
 Array = outside

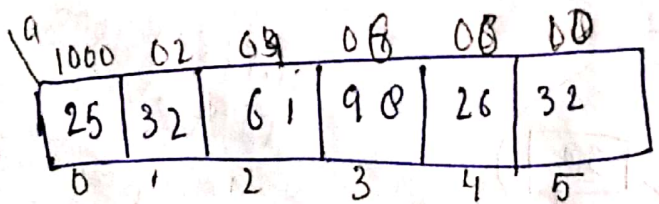
Or it can be written like this a[]
 it doesn't mean array code only base address come.

```

f(int *a, int n)
{
  if (n <= 0) return 0;
  else if (*a % 2 == 0) return *a + f(a+1, n-1);
  else return *a - f(a+1, n-1);
}
  
```

Here a+1 possible coz of variable.

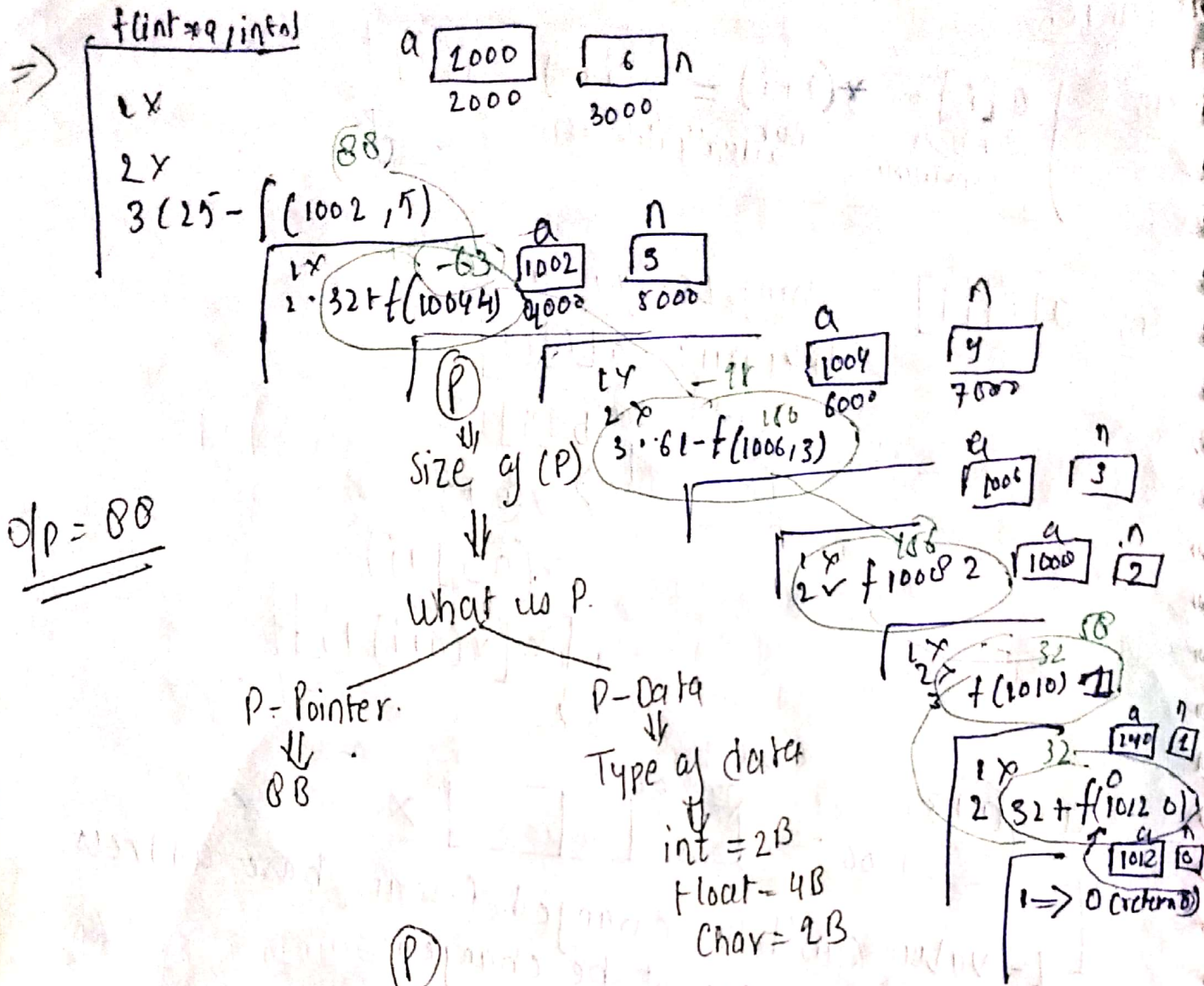
By Sir:



a \Rightarrow 1000
 a+1 \Rightarrow 1002 (coz it is a integer array. int = 2B so its skip 2 element)
 a+3 \Rightarrow 1006

a \Rightarrow 1000
 a+3 \Rightarrow 1006
 *(a+3) \Rightarrow 98
 or
 a[3]

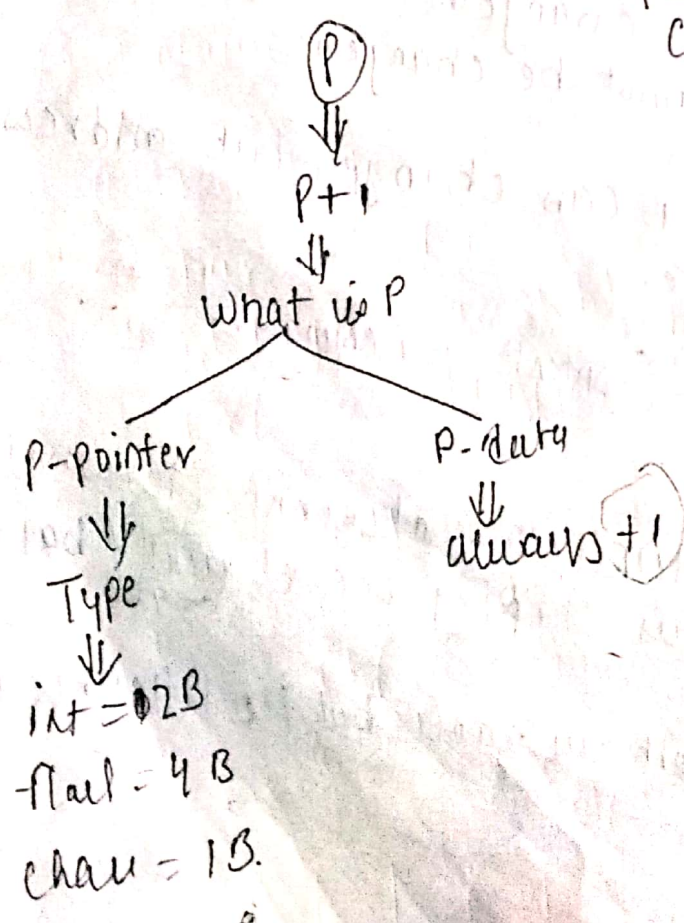
a \Rightarrow 1000
 a+1 \Rightarrow 1002
 *(a+1) \Rightarrow 32
 or a[1] = 32



$0/p = 88$

P - Pointer.
 \Downarrow
 $8B$

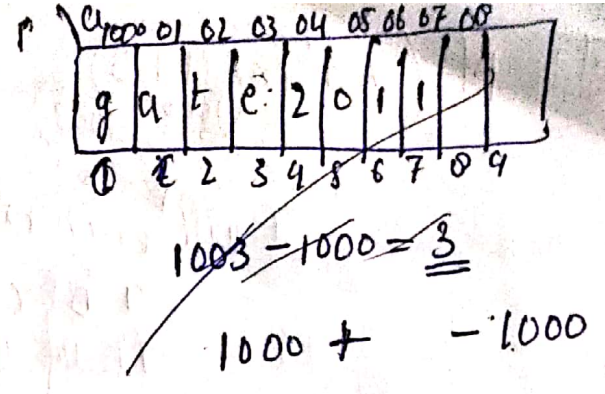
P - Data
 \Downarrow
 Type of data
 \downarrow
 int = 2B
 float = 4B
 char = 1B



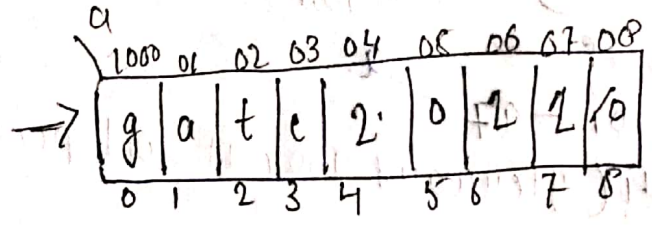
Ex: 2011 GATE
 Main()

```

{
char a[] = "gate2011";
printf("%s", a + a[3] - a[1]);
}
  
```



Every char array end with Null
 strlen("abcd") = 4
 sizeof("abcd") = 5 (include 1 Null character).
 strlen = 5



- ① $printf("%s", "gate2011");$ Gate2011
- ② $printf("%s", 2000);$ = Gate2011
give base address and it will print all the char until Null
- ③ $printf("%s", a + 3);$ e2011
1000, 1003
- ④ $printf("%c", *(a + 3));$ e
1000, 1003
- ⑤ $printf("%s", *(a + 3));$ error. X
Print string 'e' but it will print 1 char 'e' so error. Garbage value.
- ⑥ $printf("%c", (a + 3));$ Error X
Print char. but in will print 1003 string. (error) Garbage value.

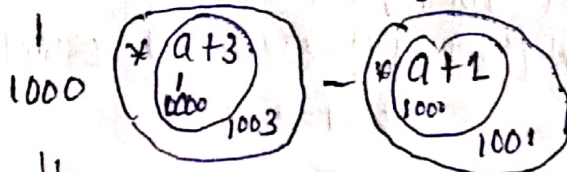
⑦ $\text{Pf}(\text{"\%d"} \times \frac{a+3}{1000}) = 101$

It will print the ASCII value.

a b c d e
97 98 99 100 101

86
0-250

⑧ $\text{Pf}(\text{"\%s"} a + a[3] - a[1]);$



\downarrow
1000 + 101 - 97

= 1004 (afterward 1004 it will print)

o/p = 2011

⑨ ex: Main()

① char a[10];

② char *b = "hello"; ^{3000 (3000 onwards all include null) = 6}

③ int length = sizeof(b); ^{5 (3000 onwards exclude null) = 5}
for (i=0; i<length; i++)

{
a[i] = b[length - i];
} ^{b[4] = short cut}

Pf("%s", a);
¹⁰⁰⁰
 $\times (b+4)$ ^{actual pointer}
³⁰⁰⁰
³⁰⁰⁴
No Output

int a[10];
sizeof(a) = 20
strlen(a) = X

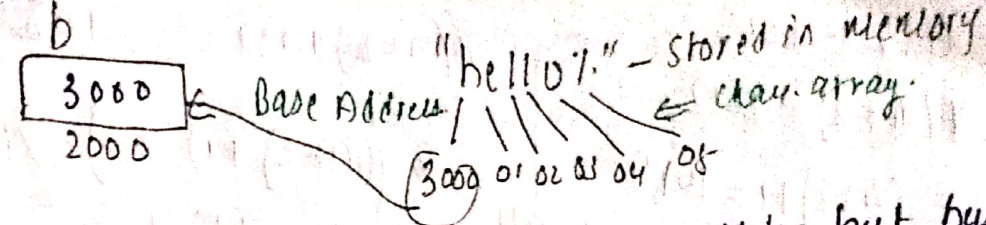
only applicable for string functions

int a[10];
char *b = "hello";
sizeof(a) = 20
sizeof(b) = 6

① =>

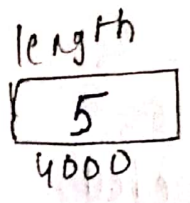
a	00	01	02	03	04	05	06	07	08	09
	0	1	2	3	4	5	6	7	8	9

②



Giving string is nothing but base address

③



$Pt(b) = 3000$
 $Pt(*b) = h$
 $Pt(*(b+1)) = e$

Main()

```
int a[5][3] = { 10, 20, 30, 40, ... 150 }
```

$Pt(" \%u \%u", a+2, *(a+2)+2) = 1016$

$Pt(" \%d", *(a+2), *(a+3)+5) = 1028$

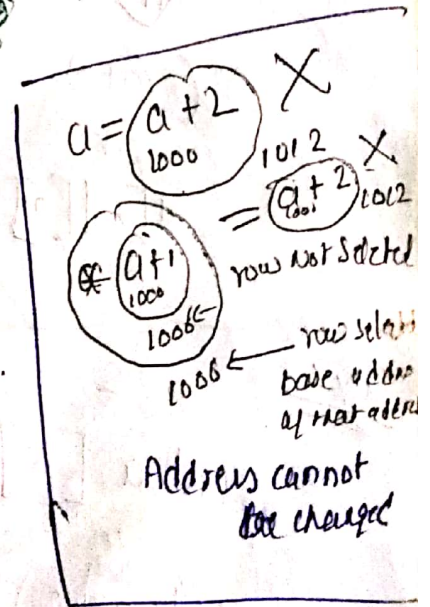
$Pt(" \%d", (a == a[0]) \&\& (*a == a[0]))$

①

a	0	1	2
0	1000 10	1002 20	1004 30
1	1006 40	1008 50	1010 60
2	1012 70	1014 80	1016 90
3	1018 100	1020 110	1022 120
4	1024 130	1026 140	1028 150

Row-major order

$a = 1000$ Address
 $a+1 = 1006$ Address
 ↳ 2D-Array skipping 1-row.
 $a+4 = 1024$
 ↳ skip $4 \times 3 = 12 \times 2 = 24$.
 ↳ 2 byte



$a = 1000$

2 stars did not get the value of 20
 $a+2 = 1012$
 $*(a+2) = 1012$
 $a+2 = 1012$
 $(a+2)+1 \Rightarrow 1018$
 $*(a+2)+1 \Rightarrow 1018$

$$a+2 = 1012$$

$$*(a+2) = 1012$$

$$** (a+2) + 1 = 1024$$

i.e (skip one element)

$$** ((a+2) + 1) = 80$$

$$a[2][1]$$

$$** ((a+2) + 1) + 2 \Rightarrow 1022$$

$$** ((a+2) + 1) + 2 \Rightarrow 120$$

$$a[3][2]$$

0 10
10 16
10 20
10 20
10 12

NOTE:

• In 3D-Array to get the data 3 star needed.
1st star. 2D skip 2D, 2nd star 2 row, 3rd get data.

• In 2D-Array to get the data 2 star needed

• In 1D " to " " " " " " " " "

$$a \Rightarrow 1000$$

$$*a \Rightarrow 1000$$

$$**a \Rightarrow 10$$



$$*(p(a)+0)$$

or

$$a[0][0]$$

$$a \Rightarrow 1000$$

$$a+1 \Rightarrow 1006$$

$$*a+1 = 1002$$

$$**a+1 \Rightarrow 11$$

$$&a \Rightarrow 1000$$

$$&a+1 \Rightarrow 1030$$

↳ skip total 2D $3 \times 5 = 15 \times 2 = 30$

Ex:

inta [5][4][7]

$$** (a+2) + 3 \Rightarrow 1034$$

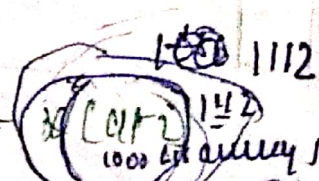
1000 1020 1020
1000
-2*2 = 4*2 = 14*2 = 28

$$** a[2] + 4 \Rightarrow 1120$$

112

two are hitting

bracket is a winner coz of highest priority. $5 \times 2 = 112$



Q1] In 2D array base address of 1-0

a[2] In 3D array base address of 2-0

a+1 = skip 1 2-D.

a+1 = selected 1-2-D.

Ex] Main ()
{

int *b[6]
// b is an array which contains 6 elements. where every one is integer pointer

1) int a[6] = {10, 20, 30, 40, 50, 60}

2) int *b[6] = {a+2, a+1, a+3, a, a+4, a+5}

3) int **c = b;

4)

*c++ (Post inc have highest priority).
Increment after.

Pf (c-b, *c-a, *c) => 1, 1, 20
 $\frac{2008-2008}{2} = 0$
 $\frac{2008-2008}{2} = 0$
 $\frac{2008-1000}{2} = 504$

Pf (c-b, *c-a, **c) => 1, 2, 30
 $\frac{2008-2008}{2} = 0$
 $\frac{2008-1000}{2} = 504$
 $\frac{1004-1000}{2} = 2$

Pf (c-b, *c-a, **c);
 1 2 31 => 1, 2, 31
 $\frac{2008-1000}{2} = 504$
 $\frac{1004-1000}{2} = 2$

1)

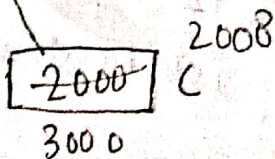
0	1000	02	04	06	08	10
	10	20	30	40	50	60

a+2 = 1000

2)

b	2000	2008	2016	2024	2032	2040
	1004	1002	1006	1000	1008	1010

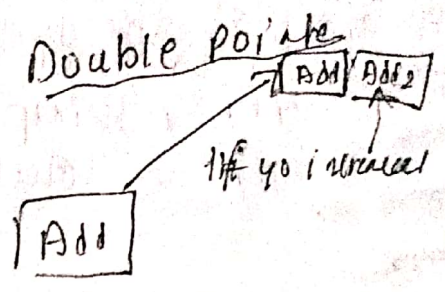
3)



4)

* & ++ post inc. both are unary operators but ++ having highest priority.

c is a integer pointer. so if pointing to it will increment by 8 byte = 2008



④ $\frac{2008 - 2000}{8} = \frac{2008 - 2000}{8} = \frac{8}{8} = 1$ element there
 B (size of 1 element is 8)
 If $2040 - 2000 = \frac{40}{8} = 5$ elements there.

NOTE 1: We can add integer constant to pointer variable.

Exp: $P + 5 =$ skipping 5 elements from P in forward dirⁿ.

$P + 5.5 =$ Not possible.

NOTE 2: We can subtract integer constant from pointer variable.

Exp: $P - 5 =$ skipping 5 elements from P in backward directionⁿ.

$P - 5.5 =$ Not possible

NOTE 3: We can subtract 2 pointers

$P_2 - P_1 =$ No of elements presents before P_2 from P_1 @

1 condition: $(P_2 - P_1)$ possible P_2 is $> P_1$

2 condition: Both should point to same array.

NOTE-4: We cannot perform Addition, Multiplication, Division between two pointers coz there is no meaning.

Ex: main()

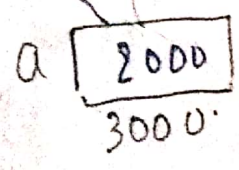
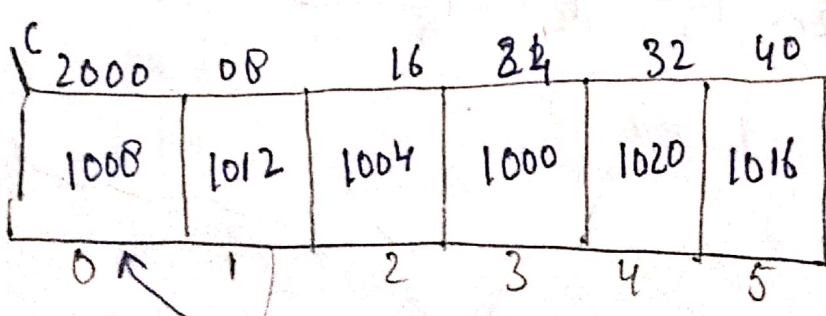
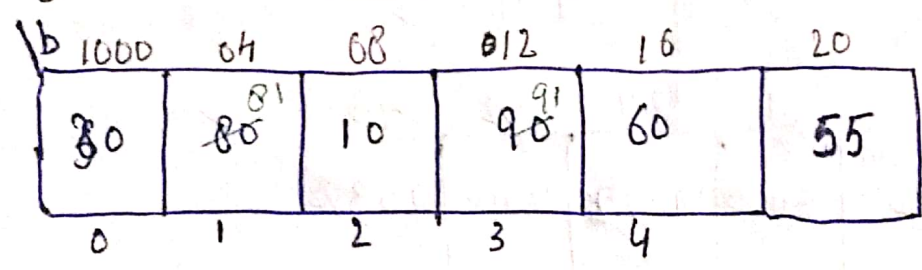
```

{
float b[] = { 30, 80, 10, 90, 60 };
float *c[] = { b+2, b+3, b+1, b, b+5, b+4 };
float **a = c;
• ++*++*a 1008 1012 91
• Pf(a-c, *a-b)
• ++**(*a+2) 2016 1008 80 = 81
Pf(a-c, *a-c) = 3
Pf(**++*a+2) = 3
}

```

O/P = 0 3
0 3
9 3

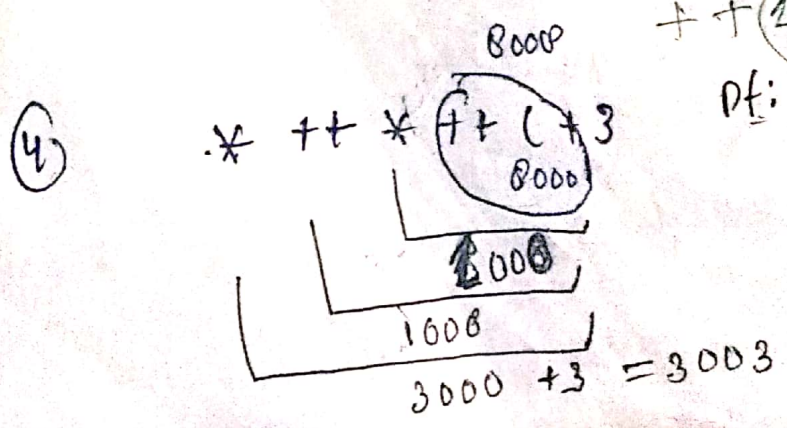
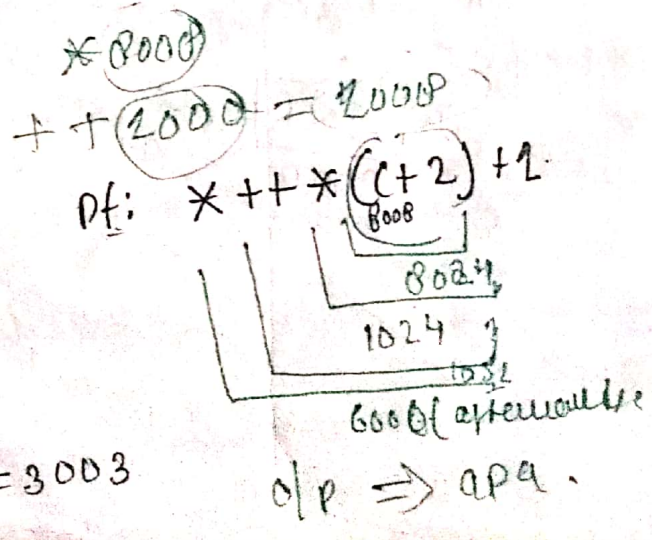
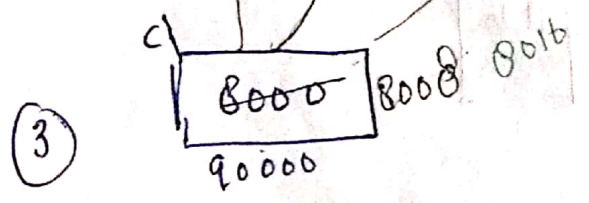
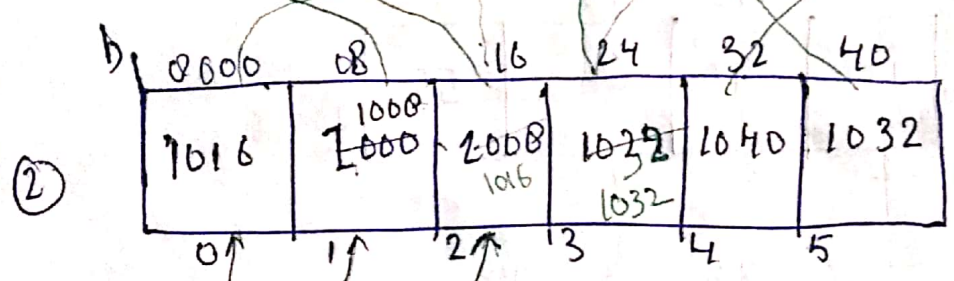
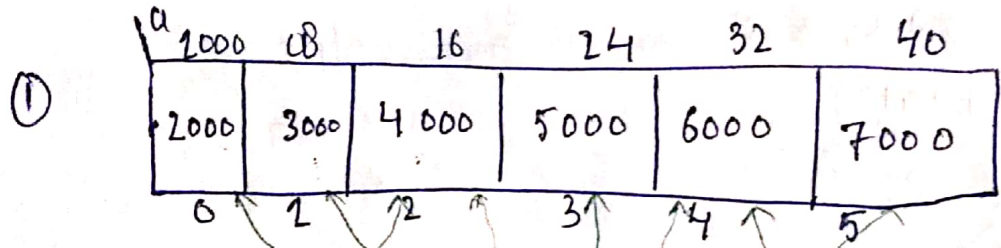
①



13/Sept Main()

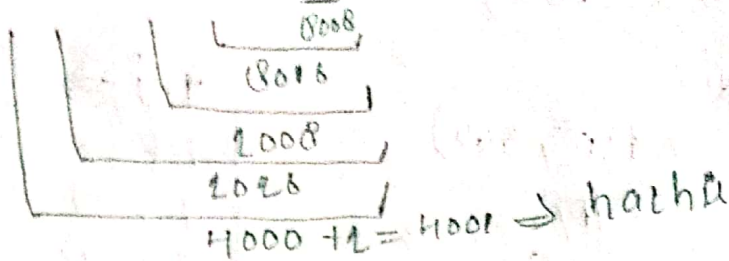
① Char *a[] = { "stupid", "baaho", "chaahi", "papa", "poo" }
 Char *b[] = { a+2, a+1, a+3, a+5, a+4 };

Char ***c = b;
 **++*++c+3;
 Pf ("%.s", **++*(c+2)+1);
 Pf ("%.s", **++*++c+1);
 Pf ("%.c", **++*(c-1)+10);
 }.



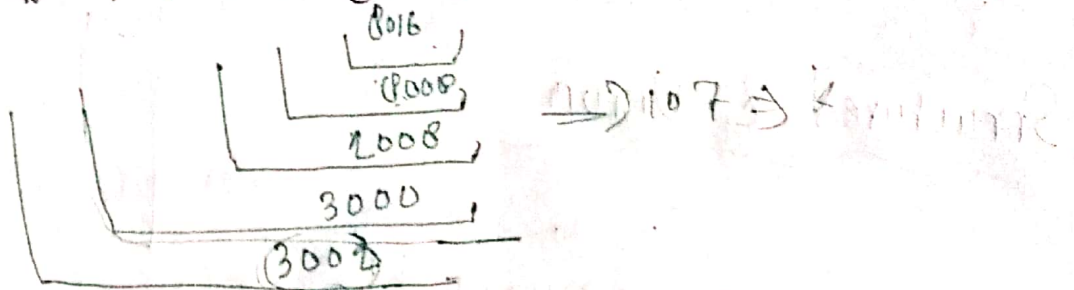
5)

* ++ * ++ C +



6)

* ++ * * (C-1) + 10



Ex:

main()

{ char a[] = "hello";

char b[] = "hello";

if (*a == *b)

 pf(hi)

 else

 pf(bi)

}

Ex:

main()

{ char *s = "%d\n";

 s++ = 2000

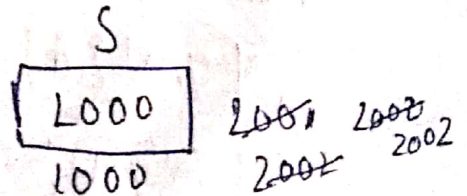
 (s++) = 2001

 p(s-2, 300);

 2002-2

 2000

 pf(2000, 300)



Pf(2000, 300)

↓ ↓

Pf("%d\n", 300)

↓ ↓
300

%u	⇒	To Print Addr
%f	⇒	" " Float
%s	⇒	" " String
%c	⇒	" " Char
%d	⇒	" " Integer

Structures & Union.

Structures: User-Defined Data type.

Ex 1: ① { main() }
struct Node.

{
int a; ⇒ 2B
float b; ⇒ 4B
char c; ⇒ 1B
} } 7B

② struct Node d = { 10, 20.5, 'a' }

Pf("%d", d.a) ⇒ 10

Pf("%c", d.c) ⇒ a

Pf("%u", &d) ⇒ 1000

Pf("%u", &(d.b)) ⇒ 1002

③ struct Node *e = &d

Pf("%u", e) ⇒ 1000

Pf("%d", (*e).a) ⇒ 10

Pre-Defined Data type d.

20
1000

② int d = 10

Pf(d) ⇒ 10

Pf(&d) ⇒ 1000

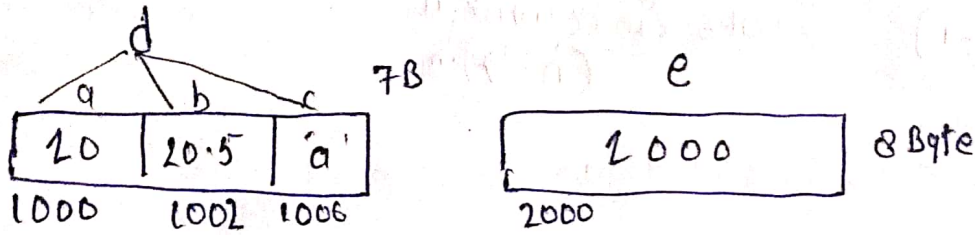
Difference

③ int *e = &d

Pf(e) ⇒ 1000

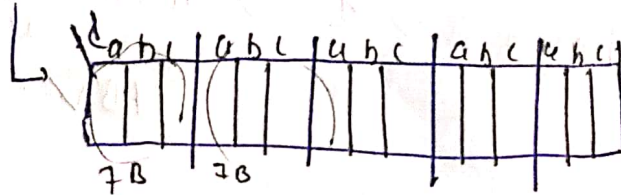
Pf(*e) ⇒ 10

Pf("%f", (*e).b) \Rightarrow 20.5
 (e $\xrightarrow{\text{or}}$ b)



Array of structure. Normally struct will be global

struct node d[5] = {10, 20.5, 'a'}



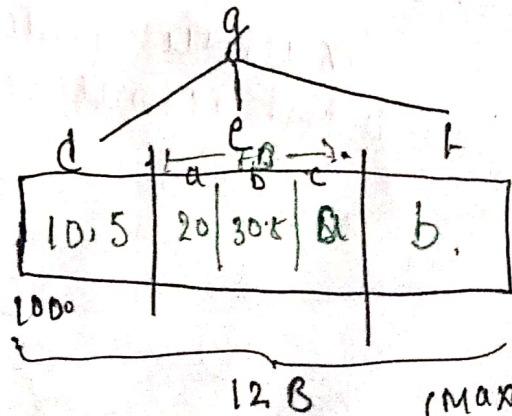
- Inside struct member not need be a pre-defined user-defined also we use.

Ex 2:

① Struct S1
 {
 int a \Rightarrow 2
 float b \Rightarrow 4
 char c \Rightarrow 1
 } 7B

② Struct S2
 {
 float d \Rightarrow 4B
 struct S1 e \Rightarrow 7B
 char f \Rightarrow 1B
 } 12B

③ struct S2 g = {10, 5, {20, 30.5, 'a', 'b'}}



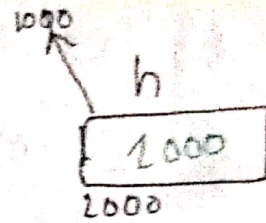
Pf("%f", g.e.b) = 30.5
 Pf("%c", g.f) = 'b'

We take
 not more than
 (max 2 dots) 2

④ Struct s_2 $\#h = \&g$

$\text{Pt} (" \%f", (\#h).e.b) = 30.5$

$\text{Pt}(h+1) \Rightarrow 1012$ can be write as $(h \rightarrow).b$



Union:

Ex: Union s_2

float d $\Rightarrow 4$

struct s₁ e $\Rightarrow 7$

char f ; $\Rightarrow 1$

};

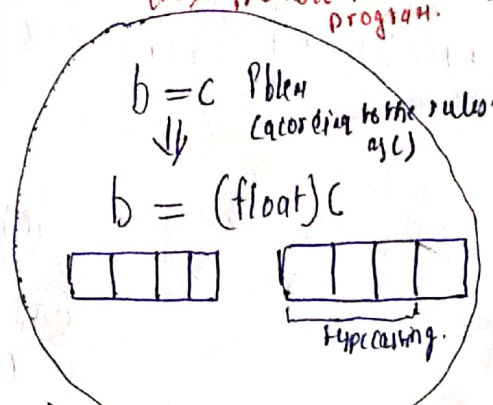
} 12 B X
instead of taking 12 B
it take larger byte of the
member.
7 B ✓

In Structure every member having their own space. but in the case of union all member will share same space but one at a time. (Normally that why we use structure instead of union because it is not clear what resource we can taking...)

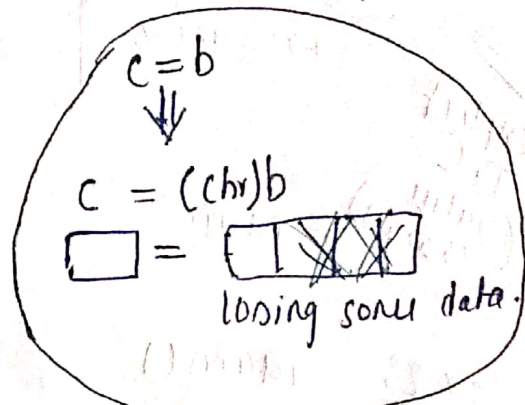
Type Casting

in Data
 int a;
 float b;
 char c;

Both should be same ^{type} if not then typecasting
 But it does trouble in program.



Done by the compiler
 Implicit T.C or coercion.

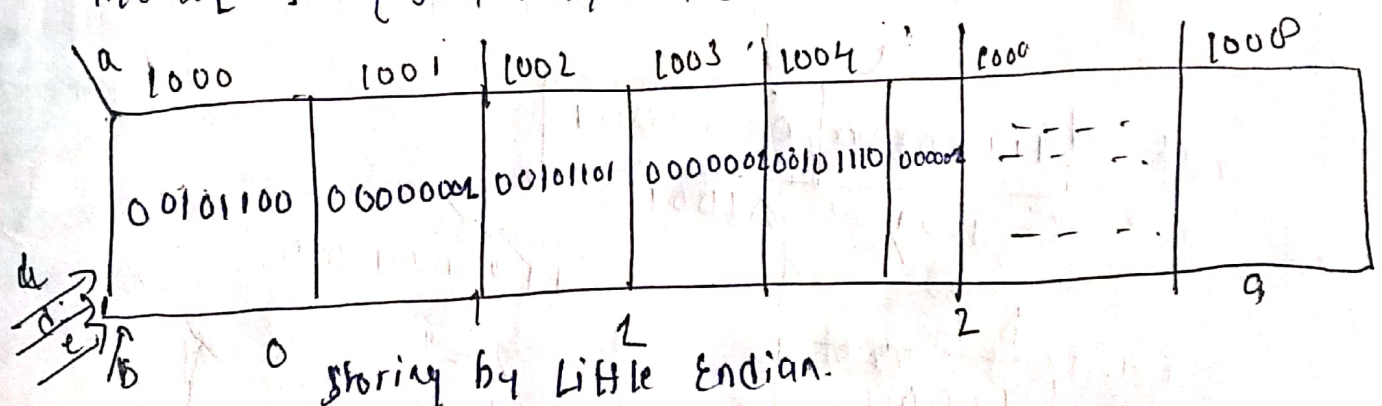


Done by the user (operator)
 explicit.

Type Casting in pointers

void - storing easy
 - retrieve difficult.
 01010100
 01011000
 = 44

int a[10] = { 300, 301, 302, 303, ..., 309 }



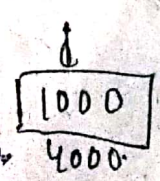
storing by Little Endian.

int *b = a

char *c = (char*)a

Type Casting

float *d = (float*)a



Type Casting.

void *e = a
 No datatype.

Type Casting (without type casting) storing.

storing easy but retrieving difficult.

$b \Rightarrow 1000$ $*b \Rightarrow 2B = 300$	$c \Rightarrow 1000$ $*c \Rightarrow 2B = 44$	$d \Rightarrow 1000$ $*d \Rightarrow 4B$ $\Rightarrow *42(\text{big data})$	$e \Rightarrow 1000$ $*e \Rightarrow \text{error}$
$b+1 \Rightarrow 1002$	$c+1 \Rightarrow 1001$	$d+1 = 1004$	$e+1 \Rightarrow \text{error}$ But retrivey

explicit
T.C
(value
changes)

~~Mat~~

$(\text{char } *c = (\text{char } *e)$

To verify in Comp
Little & Big Endian
 $\text{int } a = 300$
 $\text{char } *b = (\text{char } *)a$
 $\text{pf}(*b) = 44$

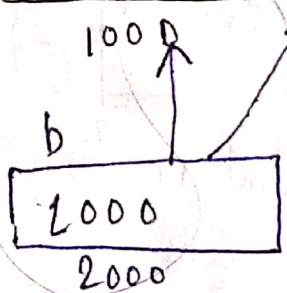
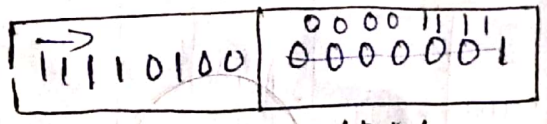
Exp 2:

```

main()
{
    int a = 500;
    char *b = (char *) &a;
    b++;
    *b = 15;
    pf("%d", a);
}

```

4084



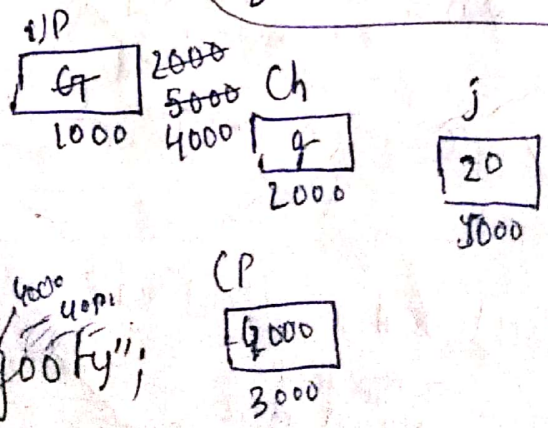
256 128 64 32 8
1 1 1 1 0 1 0 0 = 4048
 $2^2 = 4$
 $2^4 = 16$
 \vdots
 $2^{11} = 2048$

Exp 3:

```

main()
{
    void *vp;
    char ch = 'g';
    char *cp = "goofy";
    int j = 20;
}

```



```

Vp = {ch;
Pt("%c", *(int +)vp);
vp = &j;
Pt("%d", *(int*)vp); // => 20
vp = (P, 4000);
Pt("%s", ((chr*)vp + 3));
}
4000 + 3 = 4003 fy.

```

v/p: = g 20fy

ans:

```

inta a[5] = {6, 7, 8, 34, 67};
int a[4] = {23, 56, 28, 29};
int a[3] = {-12, 27, -31};
int arr[3] = {a1, a2, a3};

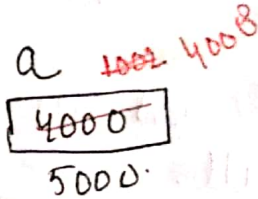
```

Using array of pointers;
 Here we have created a 2-D array in which
 1st row → contains 6 columns
 2nd row → " 4 columns.
 3rd row → " 3 columns.

```

main()
{
  f(n)
}

```

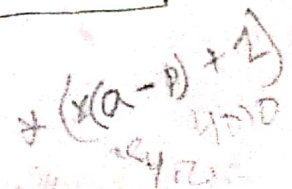
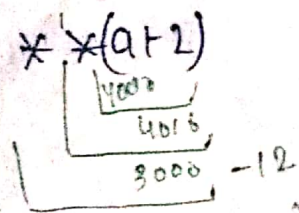
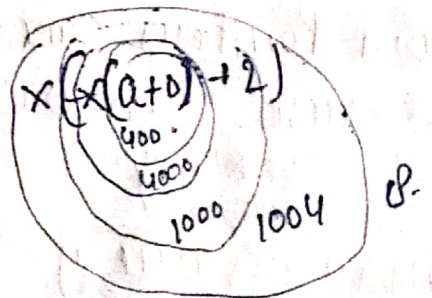


```

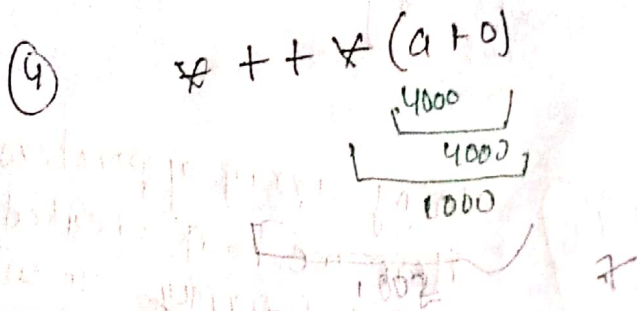
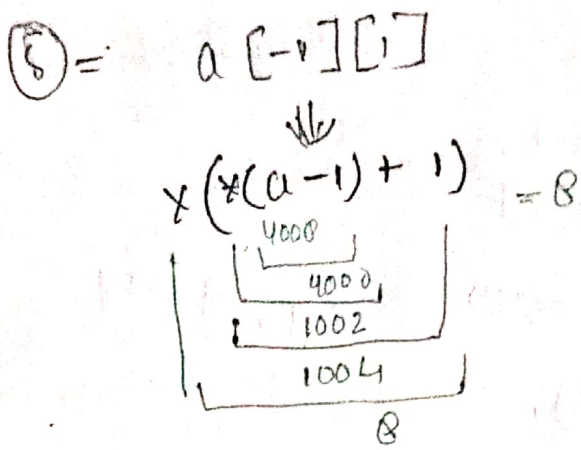
int arr[3]
f(int **a)

```

- ① Pt("%d", a[0][2]) ⇒ 8
- ② Pt("%d", *a[2]);
- ③ Pt("%d", *++a[0]);
- ④ Pt("%d", *(++a)[0]); ⇒ 7
- ⑤ Pt("%d", a[-1][1]); ⇒ 8



56



```

func() {
  auto int j=5;
  pf("%d ; j++");
}

```

```

main() {
  func();
  func();
}

```

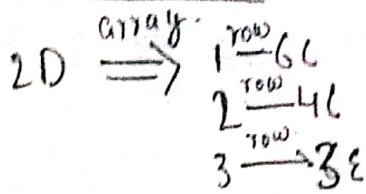
pf: 5, 5.

An object whose storage class is auto is reinitialized at every function call.

① Void Pointers cannot be used for de-referencing because each variable type takes diff. amount of memory.

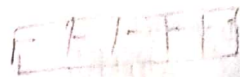
② Default value of extern storage class is 0.

int a[5][10] How it to implement 2-D array with 3 uniform row.



\Downarrow
using array pointers.

```
int a[10][10];  
int *b[10];
```



*(&a+5)+10

T/F

array of pointers it is possible.

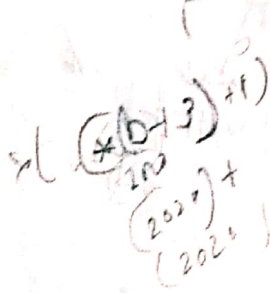
T a) $a[5][7] = \text{Value}$.

F b) $a[7] = \text{address}$. (In the middle of the array we cannot change the address)

T c) $b[3] = \text{address}$.

T d) $b[3][5] = \text{value}$.

\Rightarrow Because of array of pointers, it is possible



Linked List

Write a C program to create linked list & returning its linked list.

```

main()
{
  struct node
  {
    int data;  $\Rightarrow$  2B
    struct node *next;  $\Rightarrow$  8B
  }
}

```

```

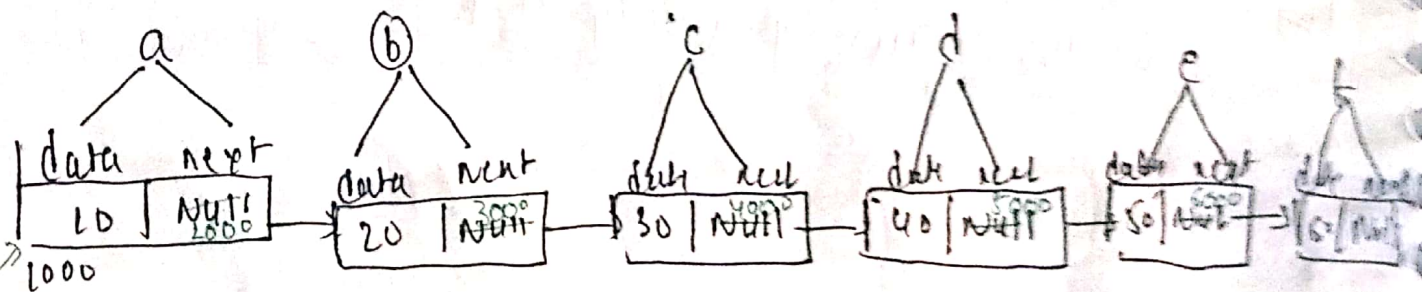
(2) struct node a = { 10, Null }
     "      b = { 20, Null }
     "      c = { 30, Null }
     "      d = { 40, Null }
     "      e = { 50, Null }
     "      f = { 60, Null }

```

```

(3) a.next = &b
     b.next = &c
     c.next = &d
     d.next = &e
     e.next = &f

```



```

(4) struct node *s = &a

```

Pf(s) \Rightarrow 1000

Pf(*s).data \Rightarrow 10

or
s \rightarrow data.

return(s)

};

Ex 2: Write a C Program. to return address of 2nd last node in the given ^{linked} list.
 ↳ linked list contain min 2 nodes

④ struct node *s = &a.

Atleast-2nd-LastNode() { function call.

;

Atleast-2nd-LastNode(struct node *s)
 {

① if (s == null)
 // empty - stop

② if (s->next == null)
 // 1-element - stop

③ while (s->next != null) {
 P = s
 s = s->next
 }
 return (P);

Best for T.C

linked list is empty. s contain null
 s or null

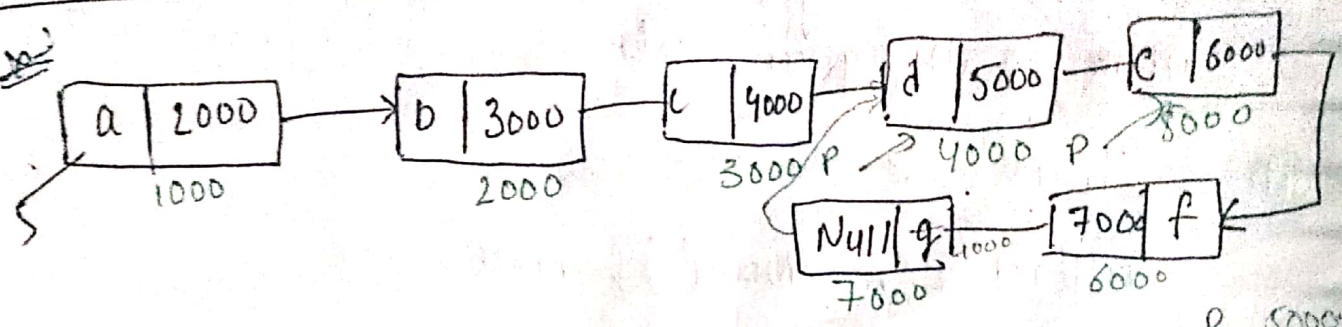
linked list is 1. s contain next null
 10 | null

Not null then this will be executed.

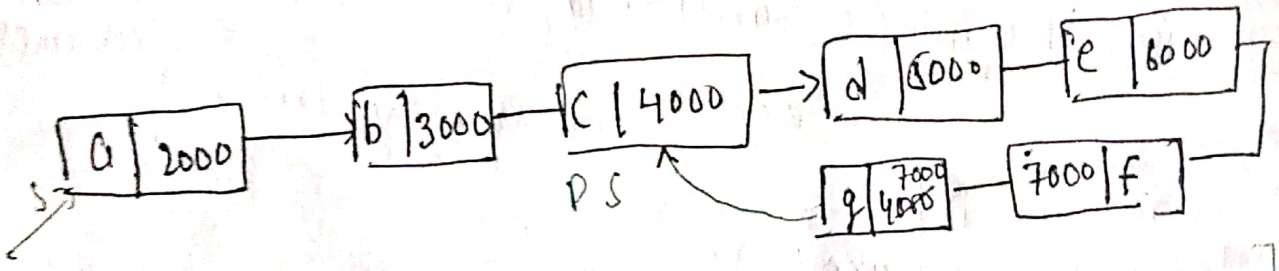
Best of Space. complexity.

14-Jept

Exo

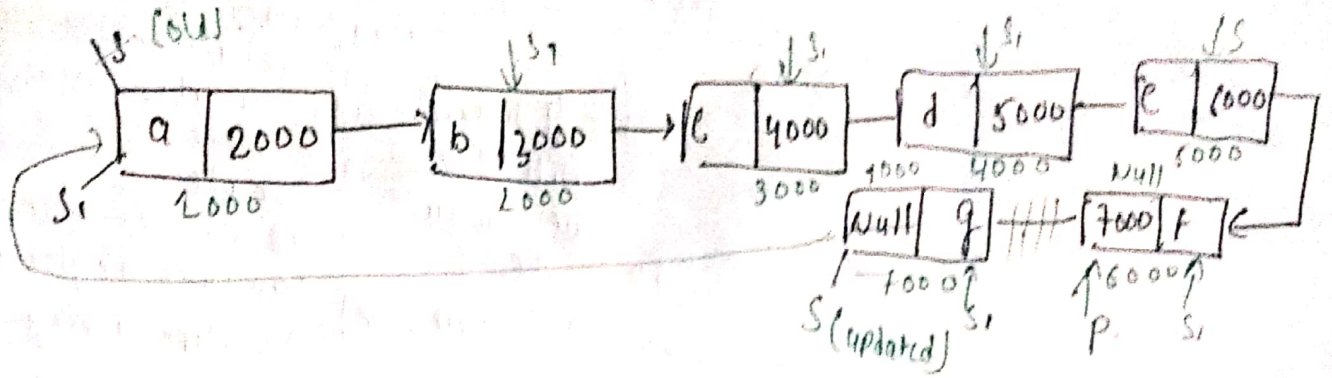


- (i) Struct node *p.
- (ii) $P = S \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (iii) $P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} = S \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (iv) $S = P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (v) $P = S \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (vi) $PF(P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{data});$



- (i) Struct node *p
- (ii) $P = S \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (iii) $P \rightarrow \text{next} = S \rightarrow \text{next} \rightarrow \text{next}; = 3000$
- (iv) $P = P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (v) $S = P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (vi) $P = S \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next};$
- (vii) $PF(P \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{next} \rightarrow \text{data});$

ans: Write a C program. to move last word of the linked list to the front of the linked list.



struct node Last-2-first (struct node *s)

function returning a address.

① if (s == Null) || (s->next == Null)
return (s);

② $S_1 = s$
while ($S_1 \rightarrow next \neq null$). // or ($S_1 \rightarrow next \rightarrow next = null$)

{
P = S_1
 $S_1 = S_1 \rightarrow next$;
}

③ $P \rightarrow next = null$
 $S_1 \rightarrow next = s$ (old s)
(updating s) $s = S_1$ } order is important.

return (s);

ans: Write a C-program. to insert a Node - with data-x at the end of the given single linked-list

insert-x-at-end (structure node *s, int x)

{
struct node *p;

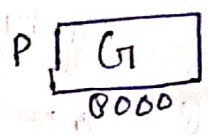
① $P = (\text{struct node}) \text{malloc}(100B)$

It will create the memory of 100B in heap area as well as returning the address also where the mem. will be create.
 Void pointer - you can store anything. So typecasting.

Diff system Diff size
 So best way to write size of (struct node)

$P = (\text{struct node}) \text{malloc}(\text{sizeof}(\text{struct node}))$
 (Type-cast) 8000 (address onwards)

if (P == Null) return(P);
 P → data = x
 P → next = null

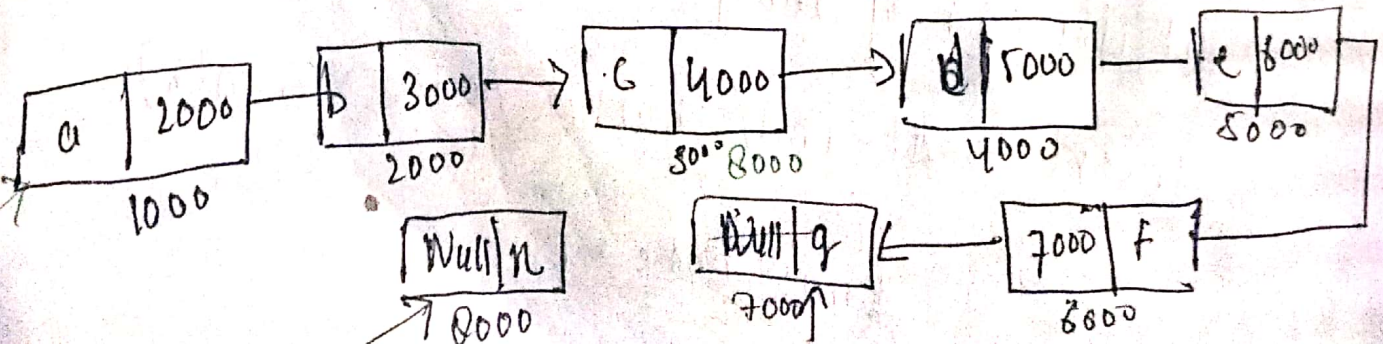


By Default Garbage value while using malloc.
 If you are using calloc instead of malloc it will clear to zero.

② if (s == Null) { s → next = Null }
 s = P
 return(s);

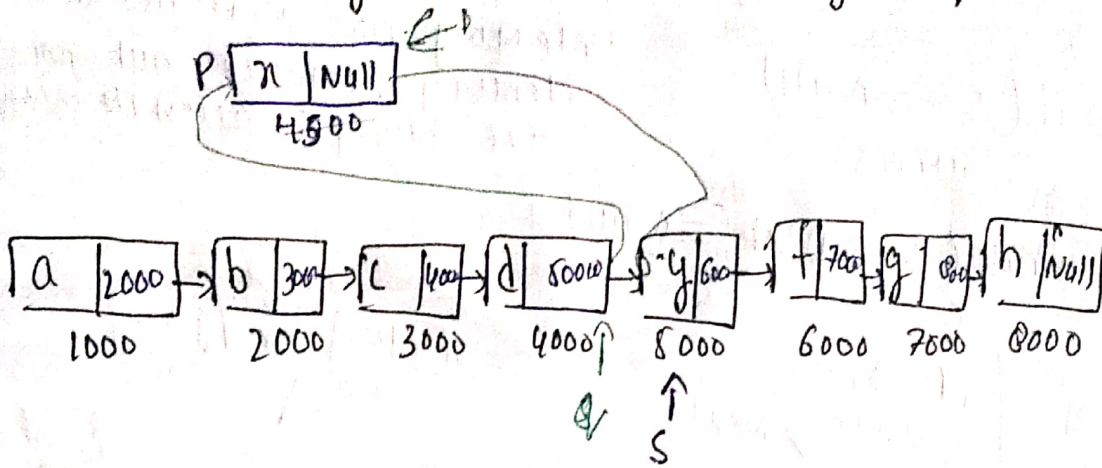
③ $S_1 = S$
 while (S₁ → next != Null)
 S₁ → S₁ → next;
 S₁ → next = P
 return(S₁);

If you want to allocate memory for elements in array.
malloc: (int*) malloc(5 * sizeof(int));
calloc: (int*) calloc(5, sizeof(int));



Inserting a node data-x before y

int b.



b
 (Strat) ~~...~~
 b
 strat
 strat
 1 q a
 a l
 y
 Strat(b) =

```
while (s->data != y && s->next != NULL)
```

```
{
    q = s;
    s = s->next;
}
```

```
if (s->data == y)
```

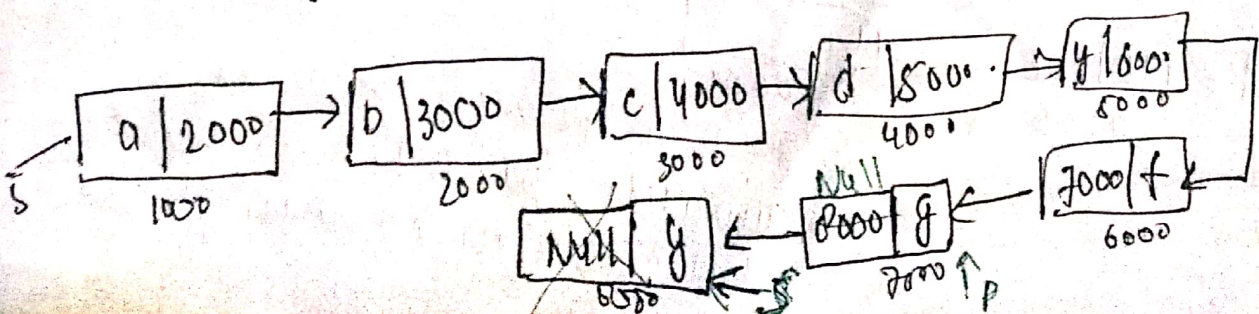
```
{
    q->next = P;
    P->next = s;
}
```

```
else
{
    Pf(No-y so-no-x)
}
```

```
if (s->data == y)
```

```
{
    P->next = s;
    s = P;
}
```

Ans: Write a C program to delete a node at the end of the given single linked list.



Delete-at-end(s)

```

{
  ① if (s == Null)
    return(s)

```

```

  ② while (s->next != Null)

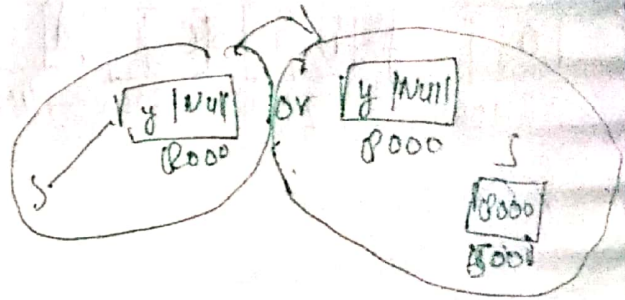
```

```

    {
      p = s;
      s = s->next;
    }

```

Segmentation error =
 ↳ Memory not available in
 Memory hierarchy but you
 are trying to access the memory



```

  ③ p->next = Null

```

```

  free(s)
  Before:
  Pt(s) = 8000
  Pt(s->data) = y
  Pt(s->next) = Null

```

```

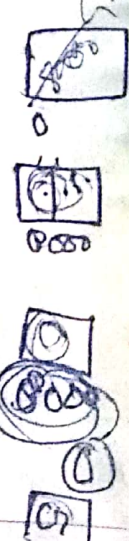
  After:
  Pt(s) => 8000
  Pt(s->data) => Garbage
  Pt(s->next) = Garbage

```

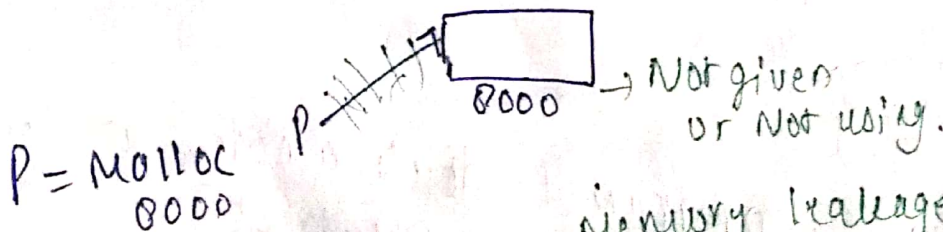
```

  Pt(s) = G
  S = Null
  Pt(s->data) => Segmentation error.

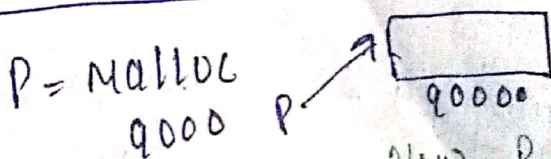
```



Dangling Pointers = Pointer to that Mem. address which is already deallocated.

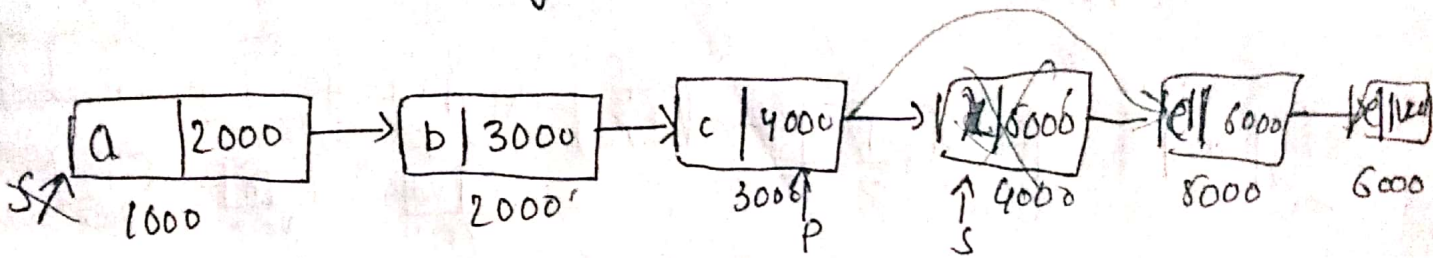


Memory leakage.



Now P is pointing to 9000
 afterwards you want to use 8000.
 but there is no way to use.

ans: Write a C Program to delete a node with data x in a given linked-list.



```

    ①  $s_1 = s$ 
    ② while ( $s \rightarrow data \neq x \ \&\& \ s \rightarrow next \neq Null$ )
        {
            {
                 $P = s$ 
                 $s_1 = s \rightarrow next;$ 
            }
            if ( $s_1 \rightarrow data = x$ )
                {
                     $P \rightarrow next = s_1 \rightarrow next$ 
                    free(s);
                     $s_1 = Null$ 
                }
        }
    
```

It is working for after one node (cos we don't know the value of P)

```

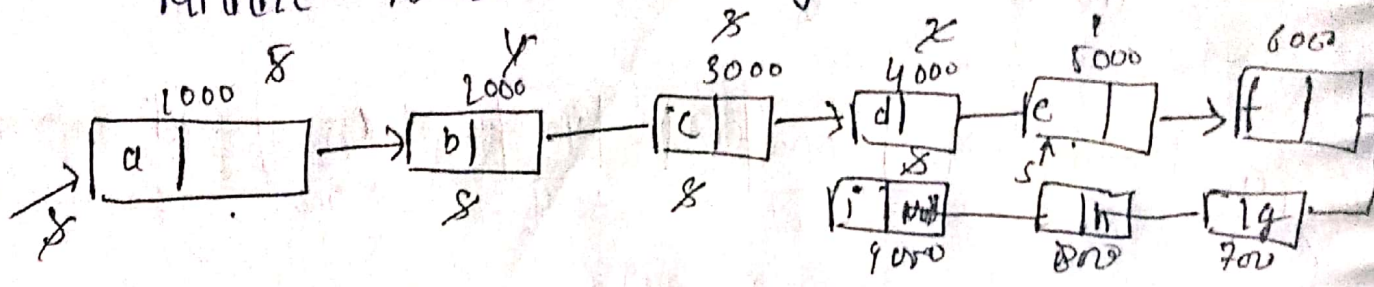
    ① if ( $s \rightarrow data == x$ )
        {
             $s_1 = s$ 
             $s = s \rightarrow next;$ 
            free(s);
             $s_1 = Null$ 
        }
    
```

If the deleting node is the first one.

① & ② are working for all the nodes in linked list.

Upto now this example is enough for Gate purpose.

Ans: Write a C-Program to find Address of Middle Node in the given linked list.



Algo 1 $C_1 = 0$ $S_1 = S$

① while ($S_1 \neq null$)

```

{
  C = C + 1
  S1 = S1 -> next;
}

```

② $C = \lfloor \frac{C}{2} \rfloor$

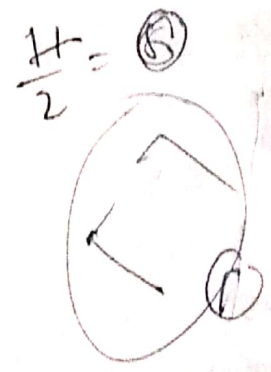
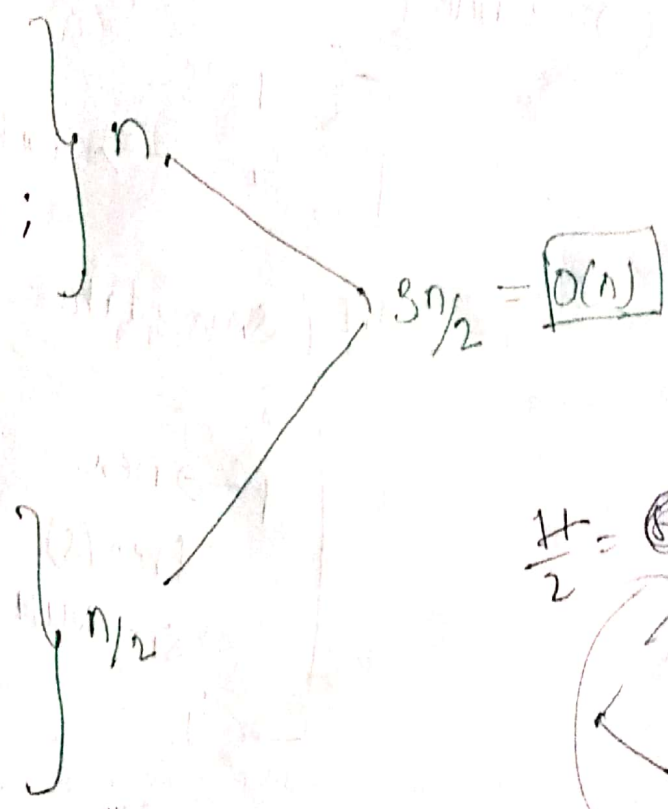
③ while ($C \neq 1$)

```

{
  C = C - 1
  S = S -> next;
}

```

return (C)



Algo 2:

P	Q
1000	1000
2000	3000
3000	5000
X1	X2

① $p = q = s$

② while ($q != null$ && $q \rightarrow next != null$ && $q \rightarrow next \rightarrow next != null$)

```

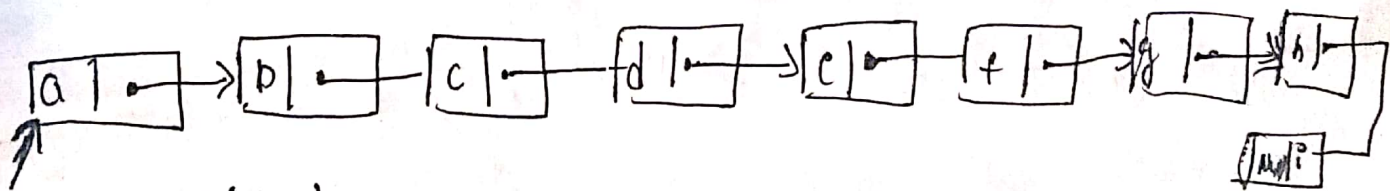
    {
        p = p -> next; // (thinking about middle) [going by 2]
        q = q -> next -> next; // (thinking about size) [going by 2]
        // fast termination condition
        // going x2 (double)
    }
    return (p);

```

⇒ O(n)

first 4r will check. after that 'q' 2-node available. then p will be inc. by 2 & q will be by 2.

Ex: Write a C-program to perform binary-search on a given linked list:



BC(S, n)

```

    {
        if (s -> next == null)
        {
            if (s -> next == x)
            {
                return (s);
            }
            else
            {
                return (NULL);
            }
        }
    }

```

```

    {
        p = mid(s)
        if (p -> data == x) return (p);
        else
    }

```

= O(n)

$$\left. \begin{array}{l} \text{else} \\ \text{if}(x < p \rightarrow \text{data}) \\ \text{else} \end{array} \right\} \begin{array}{l} p \rightarrow \text{next} = \text{null} \\ \text{BS}(q, x) \end{array} \right\} \begin{array}{l} \rightarrow \text{over} \\ = T(n/2) \end{array}$$

$$\text{BS}(p \rightarrow \text{next}, x) \Rightarrow T(n/2)$$

$$T(n) = T(n/2) + n$$

$$\left\{ \right.$$

$$\underline{O(n)}$$

Binary Search on Linked List is possible but not efficient.

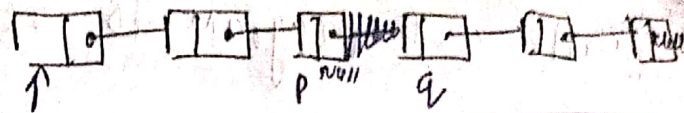
ans: Write a C program to perform Merge sort on the linked list.

MergeSort()

```

if (s → next == Null)
    return(s)
else
    {
        P = mid(s)
        q = P → next;
        P → next = Null
        S1 = Merge(s)
        S2 = Merge(q)
        S = Merge(S1, S2)
        return(s)
    }

```



Normal Merge Sort
 $2T(n/2) + n$
 $= O(n \log n)$
 space
 \downarrow
 $n + \log n$
 T both are same.

Space
 \downarrow
 $\log n$

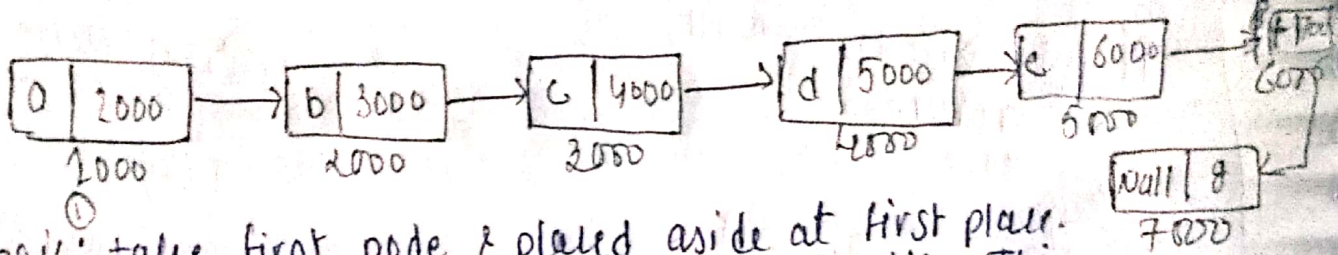
$$\frac{2T(n/2) + 2n}{2n \log n} = O(n \log n)$$

Ques: Let P be the singly linked list. What will be the complexity of the best known algorithm to check whether the linked list is palindrome or not?

- a) $O(n^2)$
- b) $O(n) \checkmark$
- c) $O(\log n)$
- d) $O(1)$

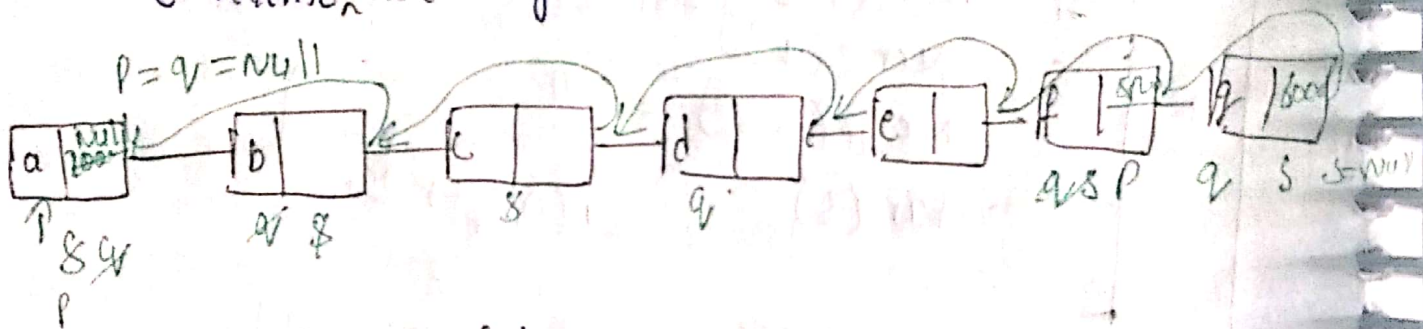
- Sol:
- 1) Find the middle element = It will take $O(1)$ time.
 - 2) Reverse the element of the list from the middle element till end.
 - 3) Compare the two halves of the list.
- This algo will take $O(n)$ time complexity

ans. Write a C-program to reverse the ^{given} single list.



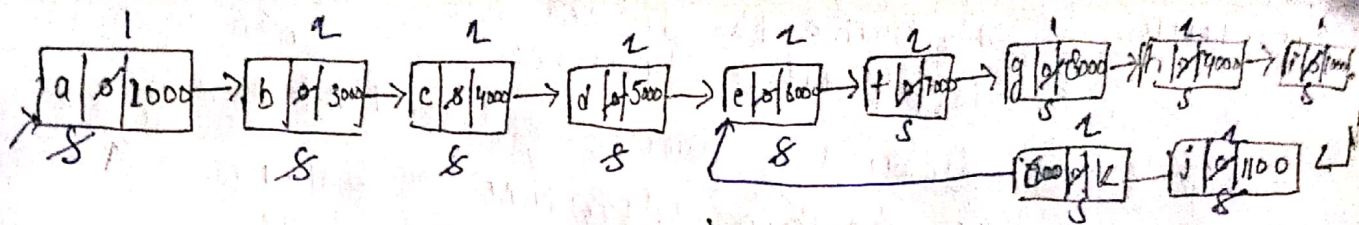
Logic ①: take first node & placed aside at first place. & then take 2nd placed at first. like this

Logic ②: ① 3 node required
② reverse ^{while} node only.



```
Reverse(s)
P = q = Null
while (s != null)
{
    P = q
    q = s
    s = s->next
    q->next = P
}
return(s)
```

ans: Write a C-program to find cycle in a given linked list.



Algorithm 1 (BFT or DFT)

- ① Assume every node contain one more extra field flag which is zero initially. using calloc
 - ② Visit every node & check flag.
 - if (flag == 0)
 - flag = 1
 - else
- PS (loop is found);

extra space
 \Downarrow
 $O(n)$

 Time $\Rightarrow O(n)$

Algorithm 2

Take pointer p & q starting initially at same place.

- ① $p = q = s$
 - ② $p = p - by - 2$ ($\times 2$)
 - ③ $q = q - by - 2$ ($\times 2$)
 - ④ while ($p \neq q$)
 - $p = p - by - 2$
 - $q = q - by - 2$
- PF (loop-found); $O(n)$

double
single row

• Intersection of two linked list of size n & n.

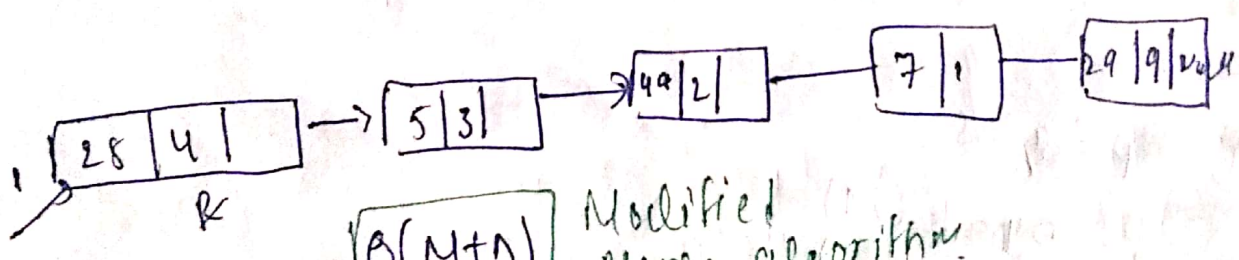
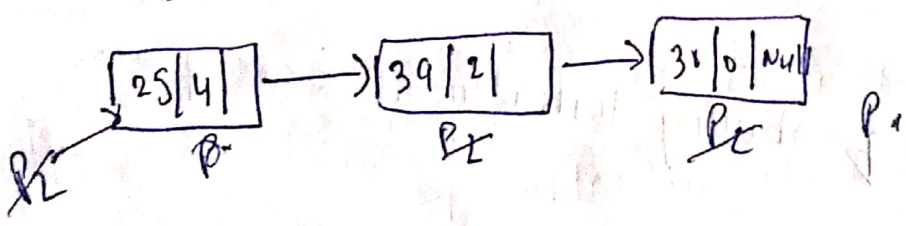
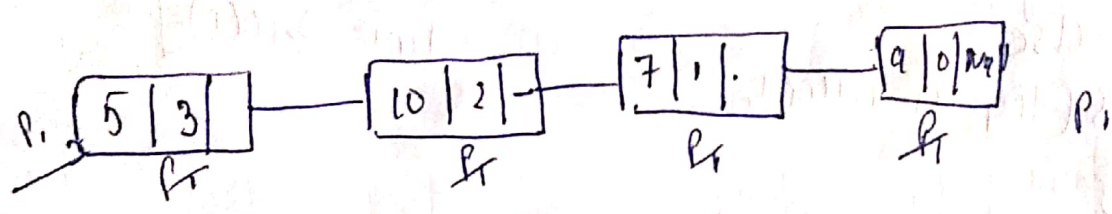
- Programme:
- Sort both the linked list. - $O(n \log n)$
 - Apply Merge Algorithm. - $O(n^2)$
 - Similar union of two linked - also, $= O(n \log n)$.
 - Membership Algorithm of linked list.
 - Circularity Algo - for all time.

This expression is possible using Polynomial Addition using L.L.

Qp: $P_1: 5x^3 + 10x^2 + 7x + 9$
 $P_2: 25x^4 + 39x^2 + 30$

Comparing which is greater than take it based on the power.

$P_1 + P_2: 25x^4 + 44x^2 + 7x + 39$

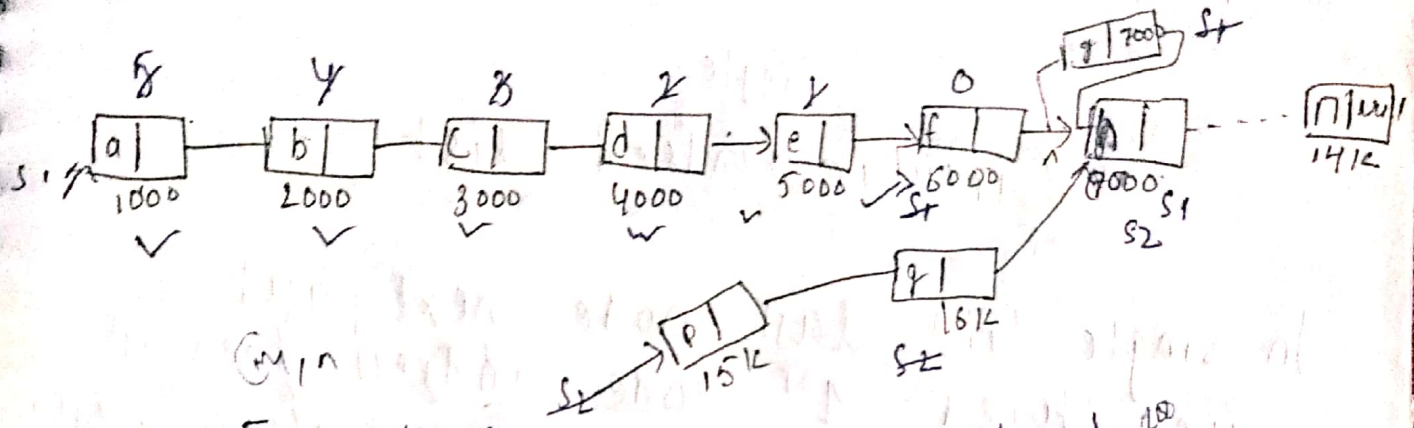


$O(M+N)$ Modified Merge algorithm.

Polynomial Multiplication using linked list will take $O(M \times N)$ time.

Drawbacks of Single List:

Ques = S_1 contain M -elements S_2 contain n -elements
to find 2nd intersecting node address. to find
How much time?



① Difference = $M - n$
 $\begin{matrix} 5 & 14-9 \\ & L_{max} \end{matrix}$
 (S1 move up to 5 place)
 M is greater by 5.
 First element the greater one & then onwards.

② while (D != 0)

```

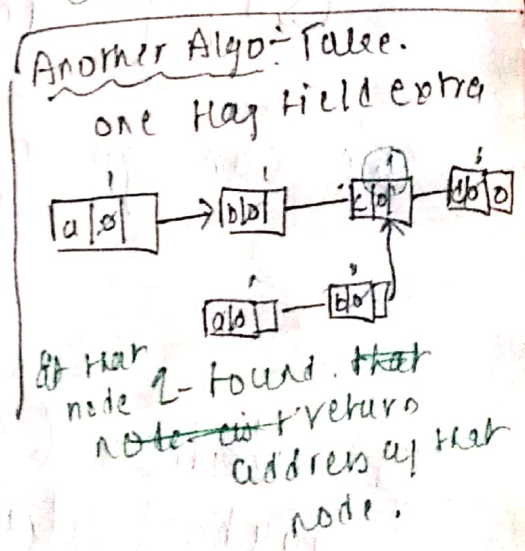
    {
        S1 = S1 -> next
        D = D - 1
    }
    
```

while (S1 != S2)

```

    {
        S1 = S1 -> next
        S2 = S2 -> next;
    }
    
```

$O(m)$
 $\max(M, n)$



Drawbacks of Single linked list;

- ① We cannot go back. Because every node contains next node address. so we cannot go back.
- ② Single linked list last node next part always null that means we are

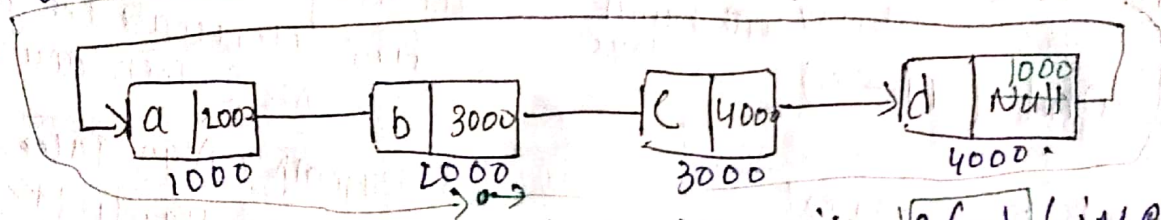
not utilizing properly.

To eliminate above drawback we are going to use Circular linked list.

Single.

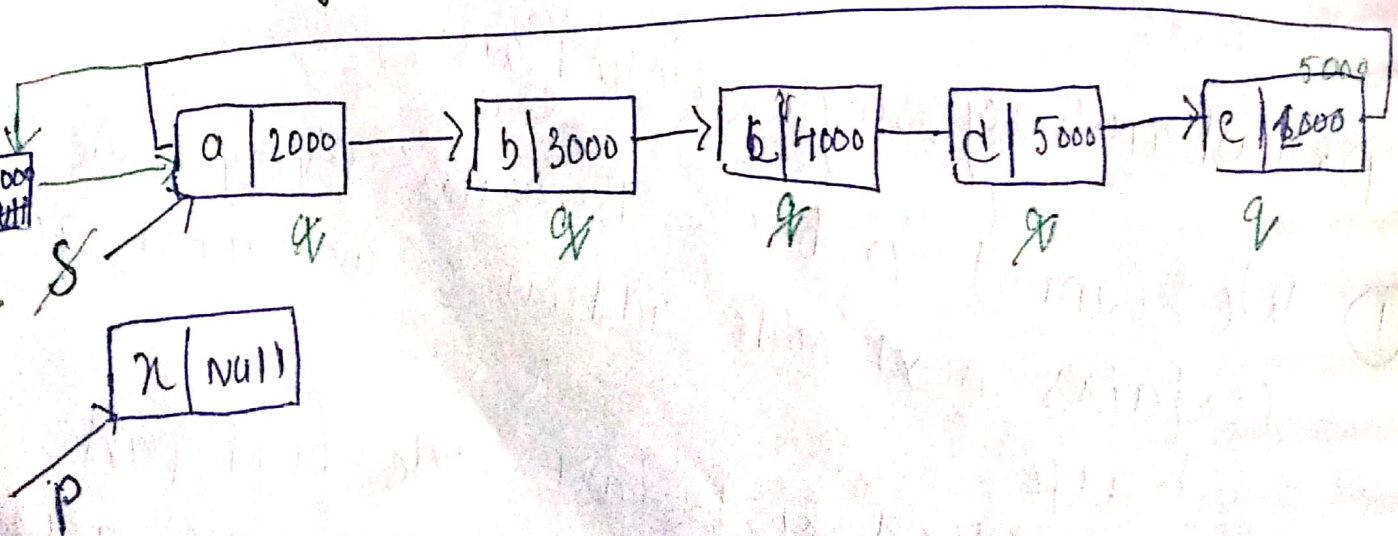
Circular linked list

In single list last node next part it we stored 1st node address then it is known as circular single linked list.



We can go back but it will $O(n)$ time.
Possible but it will take more time

Ques: Write a C-Program to Insert a Node at the start of the given circular linked list.



① $P = \text{malloc}$

② $q = s$

while ($q \rightarrow \text{next} \neq s$)

$q \rightarrow q \rightarrow \text{next}$

③ $P \rightarrow \text{next} = s$

$q \rightarrow \text{next} = P$

$s = P$

return(s);

Time Complexity: $O(n)$

Delete Last Node in Circular Single linked list:

① $P = s$

while ($P \rightarrow \text{next} \neq s$)

{
 $q = P$
 $P = P \rightarrow \text{next}$
}

$q \rightarrow \text{next} = s$

free(P)

$P = \text{Null}$

return(s)

Double Linked List: It will take less time
But taking more space

Struct DLL

{

struct DLL *prev;

int data;

struct DLL *next;

} 18B.

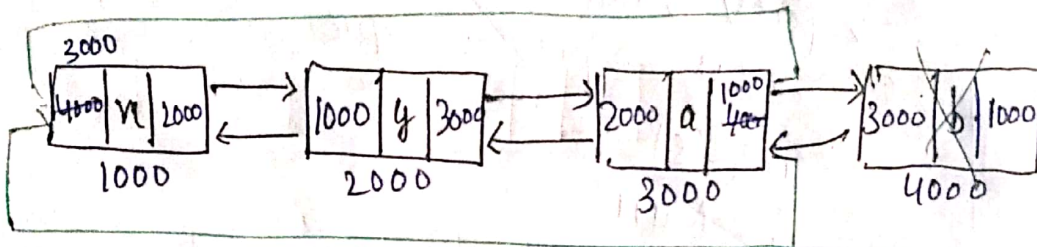
[Any node previous null then
it is 1st node or any node
next null is last node]

$P \rightarrow \text{prev} \rightarrow \text{next} = P.$

Add - n - begin.

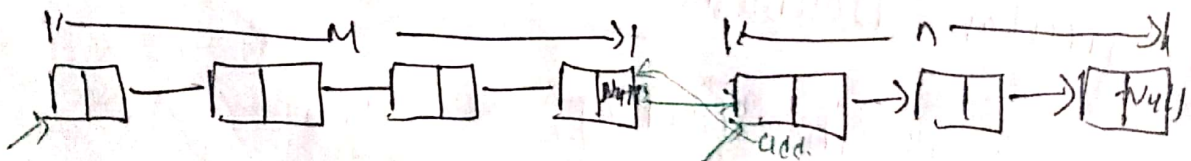
Deleting Last Node:

$P = S \rightarrow \text{prev};$
 $P \rightarrow \text{prev} \rightarrow \text{next} = S$
 $S \rightarrow \text{prev} = P \rightarrow \text{prev}.$
 $\text{free}(P)$
 $P = \text{Null}$



Concatination : (S_1, S_2) : S_1 followed S_2 (End of S_1 should be connected to first node of S_2).

SLL

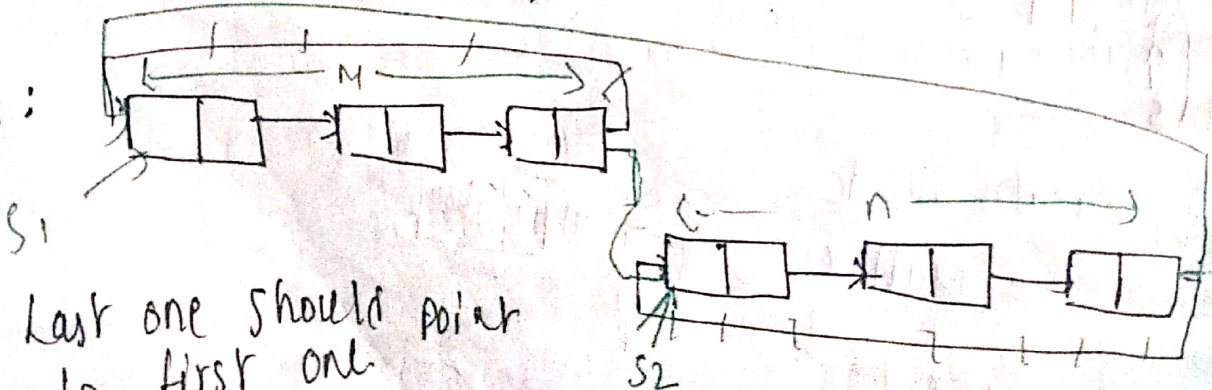


S_1

T.C: $O(M) [EC]$

S_2

C.S.L

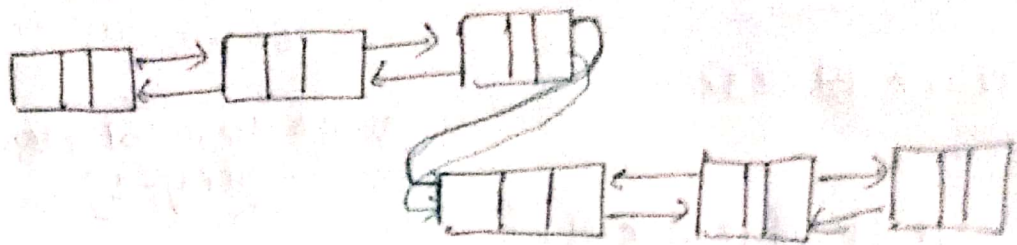


Last one should point to first one.

S_2 should point to S_2

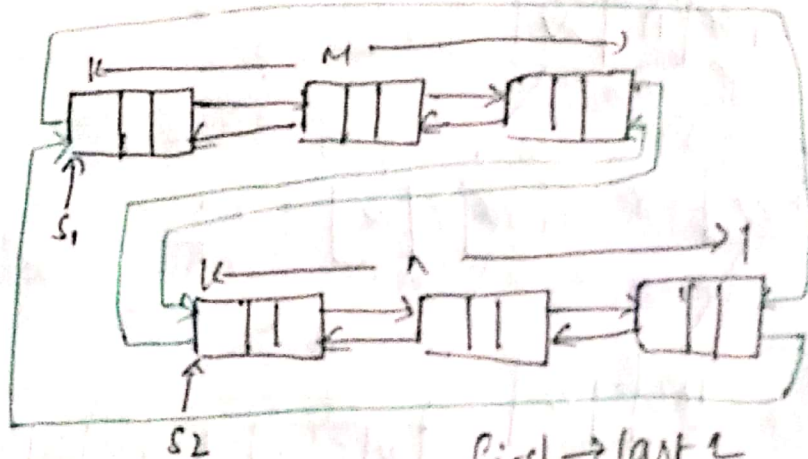
T.C: $(O(M+N)) [EC]$

DLL:



$T.C = O(N)$ ϵC

C-DLL:



$T.C: O(N)$ ϵC

first \rightarrow last ϵ So $O(N)$.
Last \rightarrow first ϵ

NOTE: The Greatest Advantages with a Double or Circular Double Linked list is if we lost one pointer. Then with the help of another pointer we can recover it or we can get it back.

Binary Trees:

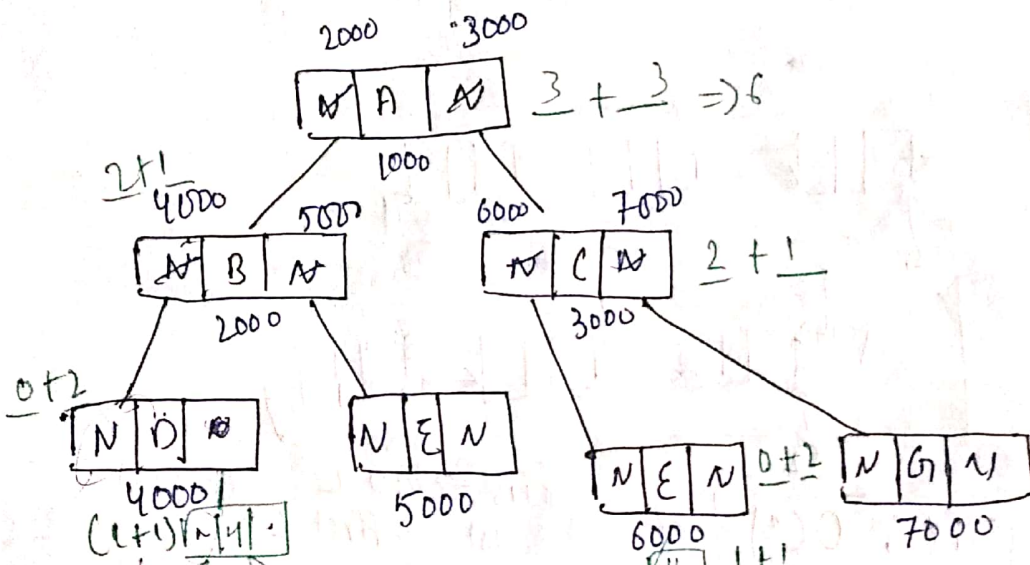
Linked List are not suitable for Binary search.

Struct BNode

```

{
  struct BNode *lc;
  int data;
  struct BNode *rc;
}
  
```

No. of nodes in a BT of degree two is $= \text{leaf} - 1$



ans: Write a C-program to count number of leaf nodes in the given binary tree.

```

leaf count
count
  (NLF (struct BinaryT *r) => T(n))
  {
    ① if (r == null) return(0) // If the tree is empty & no leaf & no root
    ② if (r->lc == null && r->rc == null) return(1) // Doubt
  }
  
```

```

else
  a = (NLF(r->lc)) => T(n/2)
  b = (NLF(r->rc)) => T(n/2)
  c = a + b; } c
  
```

Non-leaf
not count

return(0);
}

$$2T(n/2) + c$$
$$= \boxed{O(n) [EC]}$$

↳ Every node you have to ask (are you leaf node or not atleast 2 times).

Stack space
↓
Min $\Rightarrow \log n$
Max $\Rightarrow n$.

If you are counting non-leaf nodes
then — if (r->l == null || r->r == null) .
if part (return(0))
else
c = 1

17/Sept/2019

Write a program to Count Number of Internal-Nodes
or non-leaf nodes. in its Given binary tree.

CNIN(R)

① if (R == Null)
return(0)

② if (R->next == Null || R->r == Null)
return(0).

a = CNIN(R->l)

b = CNIN(R->r)

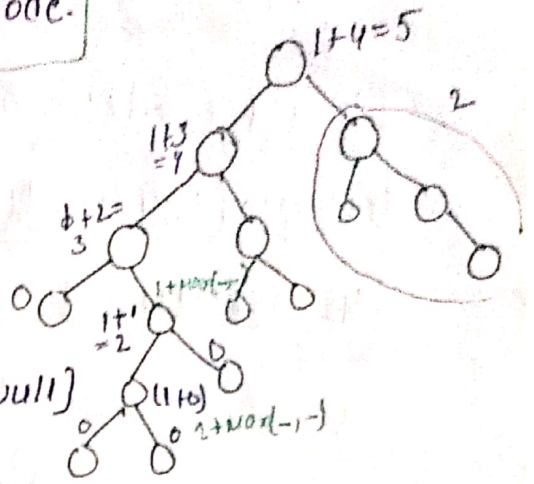
Height of BT = Height of root Node.

HOBT(R)

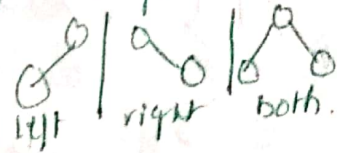
- ```

{
 ① if (r == Null)
 return (0);
 ② if (r->lc == Null && r->rc == Null)
 return (0);
 else
 ③ a = HOBT(r->lc)
 b = HOBT(r->rc)
 c = 1 + max(a, b)
 return (c)
}

```



Because of non-leaf one edge always there.



If you want to find No of levels in Binary Tree.

- ```

② if (r->lc == Null && r->rc == Null)
    return (0); or Height of BT + 2

```

T.C: $O(N)$ [EC]

Program: Write a C-program to verify Given Binary Tree. is strict Binary tree or not.

Strict Binary Tree: Every node having 0 or two children.

SBT(r)

- ```

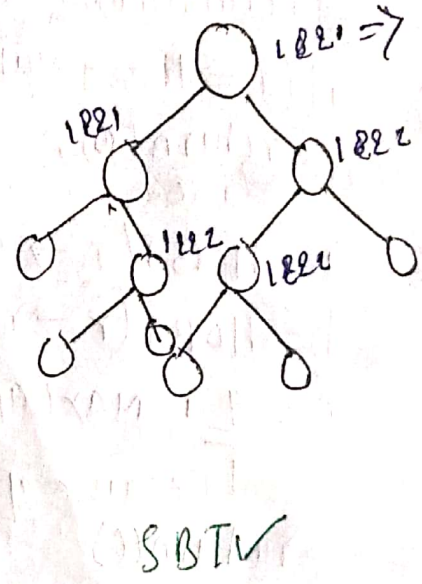
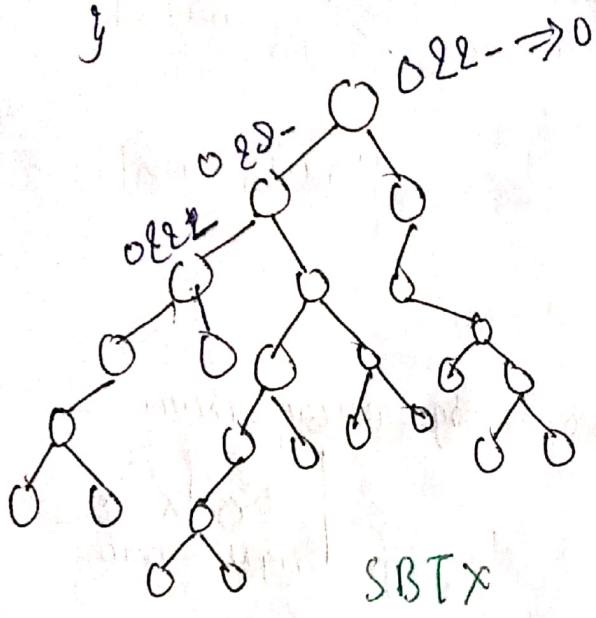
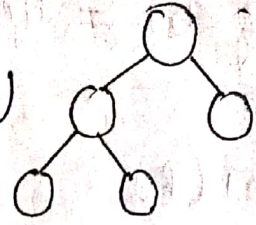
{
 ① if (r == Null)
 return (1);
 ② if (r->lc == null && r->rc == null)
 return (1);

```

```

else
3. if (r->lc != null && r->rc != null)
return (SBT(r->lc) && SBT(r->rc))
4. return (0). (Non-leaf fail return 0, no need to
check further).

```



Write a C-Program to verify given two-Binary Tree are equal or not.

```

=> V2BT(r1, r2)
1. if (r1 == Null && r2 == Null)
return (1);
else
return (0);
2. if
3. if ((r1 == null && r2 != null) || (r1 != null && r2 == null))
return (0);
4. if (r1->data == r2->data)

```

return( VZBT(r<sub>1</sub> → dl, r<sub>2</sub> → lc) || VZBT(r<sub>1</sub> → rc, r<sub>2</sub> → rl)

return(0);

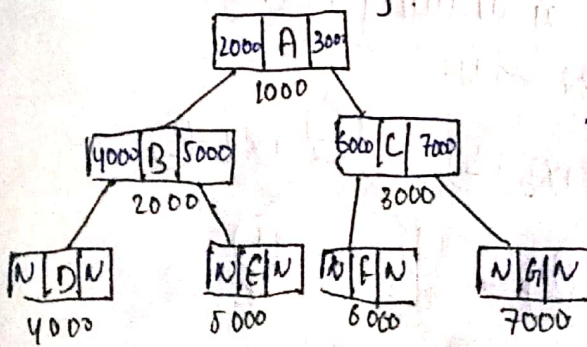
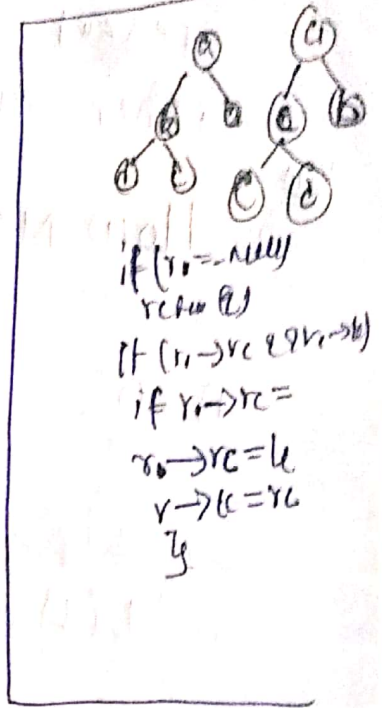
};

ans: Write a C Program to Convert Binary tree into its mirror image.

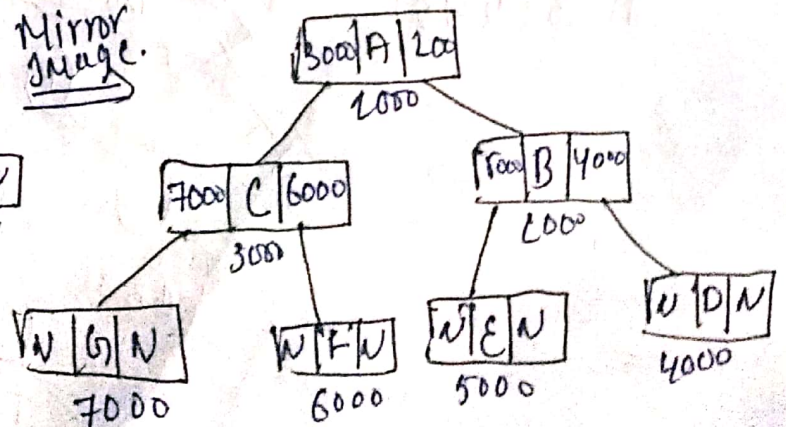
```

Mirror-Image(r)
{
 ①: if (r == NULL) [nonode & empty]
 return(0);
 ②: if (r → rc == NULL && r → dl == NULL);
 return(0). [only one node]
 ③ else. [contain. non-leaf node]
 {
 if (r → rc != NULL && r → dl != NULL)
 {
 r → rc = dl;
 r → dl = rc;
 t = r → rc;
 r → rc = Mirror-Image(r → dl);
 r → dl = Mirror-Image(t);
 }
 return(r);
 }
}

```



Mirror Image.



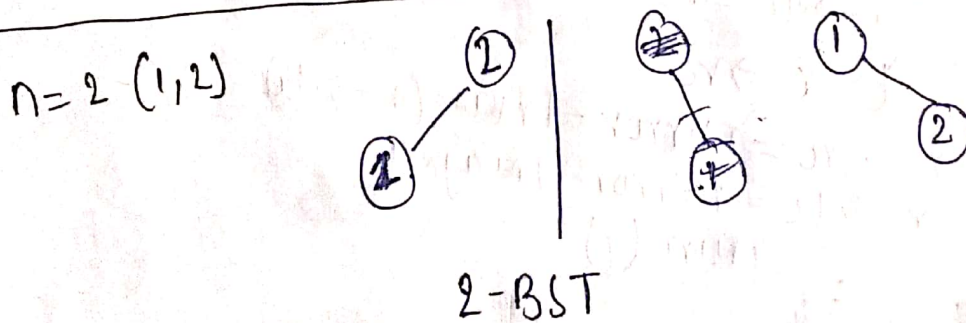
Binary Search ~~Order~~ Tree (BST). left = smaller  
right = greater.

⇒ In the given Binary tree. at every node. Comparing root data all elements present on the left sub tree, <sup>is</sup> smaller. & all elements present on the right <sup>is</sup> side is greater. then it is known as BST.

⇒ How many BST's possible with n-nodes.

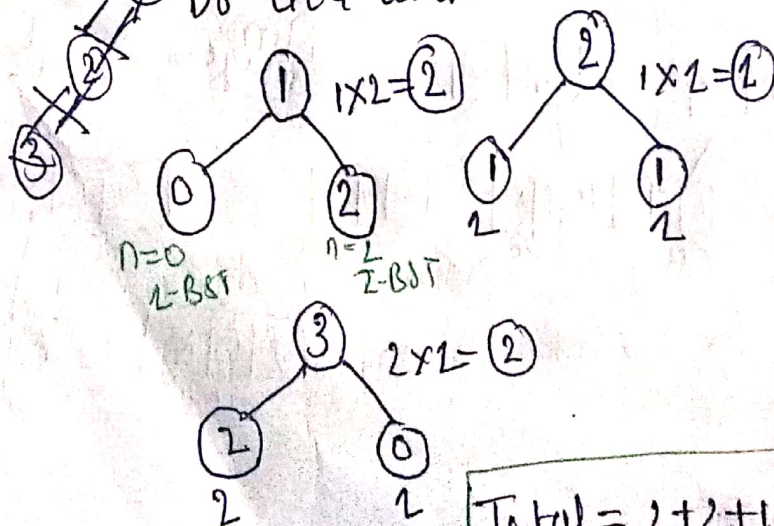
n=0      2-BST  
Empty Set

n=1 (1)  
BST

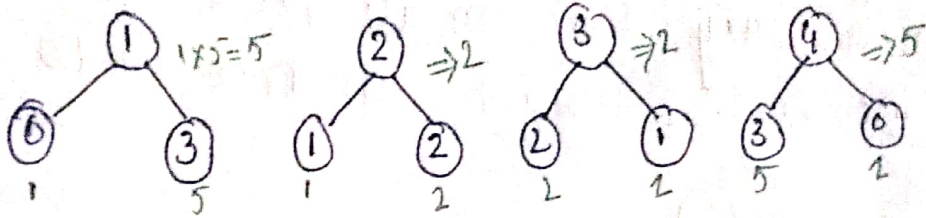


n=3 (1,2,3)

Instead of drawing manually 5 BST  
Do like this:

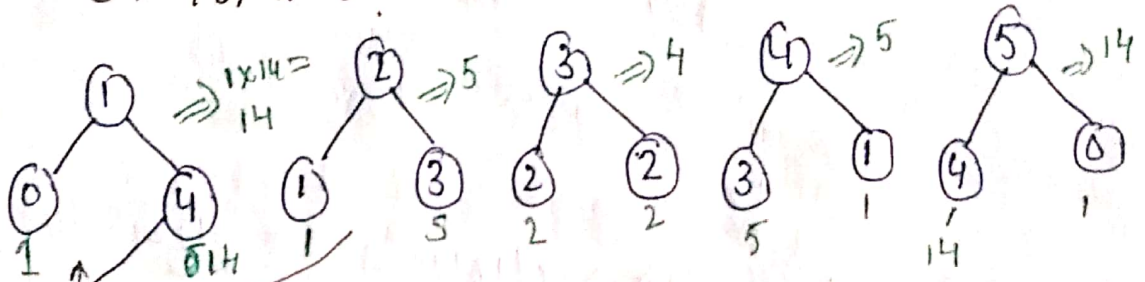


$$n = 4(1, 2, 3, 4)$$



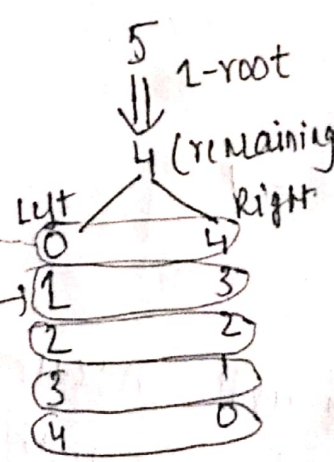
$$\text{Total} = 5 + 2 + 2 + 5 = 14$$

$$n = 5(1, 2, 3, 4, 5)$$



$$\text{Total: } 14 + 5 + 4 + 5 + 14 = 42$$

indicate with h node how many BST possible.



$$BST(N) = \sum_{L=0}^{N-1} BST(L) \cdot BST(R)$$

$$L + R = N - 1$$

$$R = N - (L + 1)$$

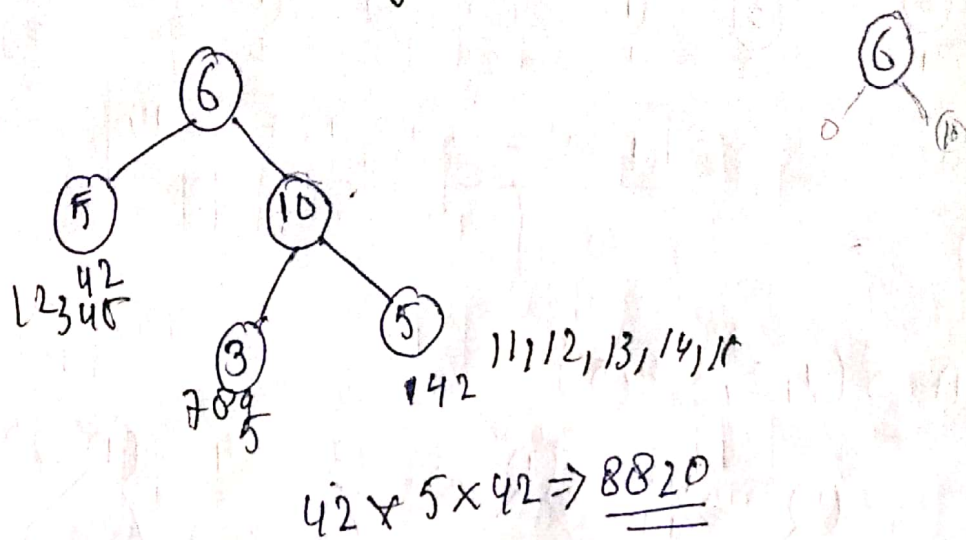
or you can use =

$$\frac{2n C_n}{n+1}$$

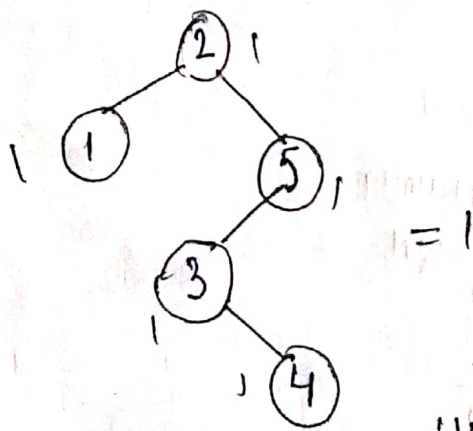
$$\frac{10 C_5}{6} \Rightarrow$$

$$\frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5 \times 6} = 10$$

Ques: How many BST possible with 15 nodes (1, 2, ..., 15) in such a way that in all those BST 6 will be the root & right sub tree is 10.



Ques: Doubt  
 i/p: Set S with 5-elements  $S = \{1, 2, 3, 4, 5\}$   
 unlabelled Binary-Tree with 5-nodes.



Ques: No. of way we will insert data from Set S into U.L.B. so that it becomes BST.

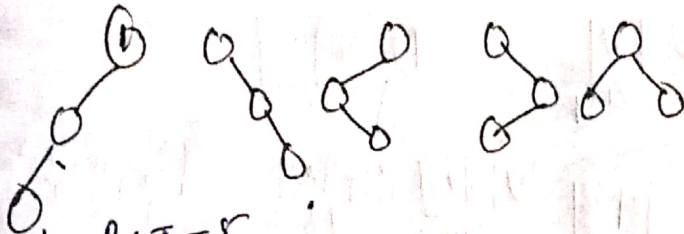
Ans = 1-way

NOTE: No. of BST<sup>s</sup> with n-nodes = No. of Unlabelled BST<sup>s</sup> with n-nodes. =  $\frac{2^n C_n}{n+1}$

$\Rightarrow$  No of Binary tree = No<sup>n</sup> of BST's  $\times n!$

$n=3$

No of unlabelled BST possible (5).



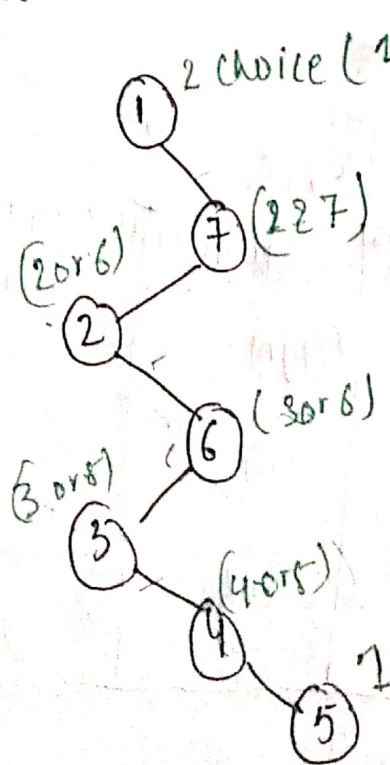
BST = 5

UBST = 5

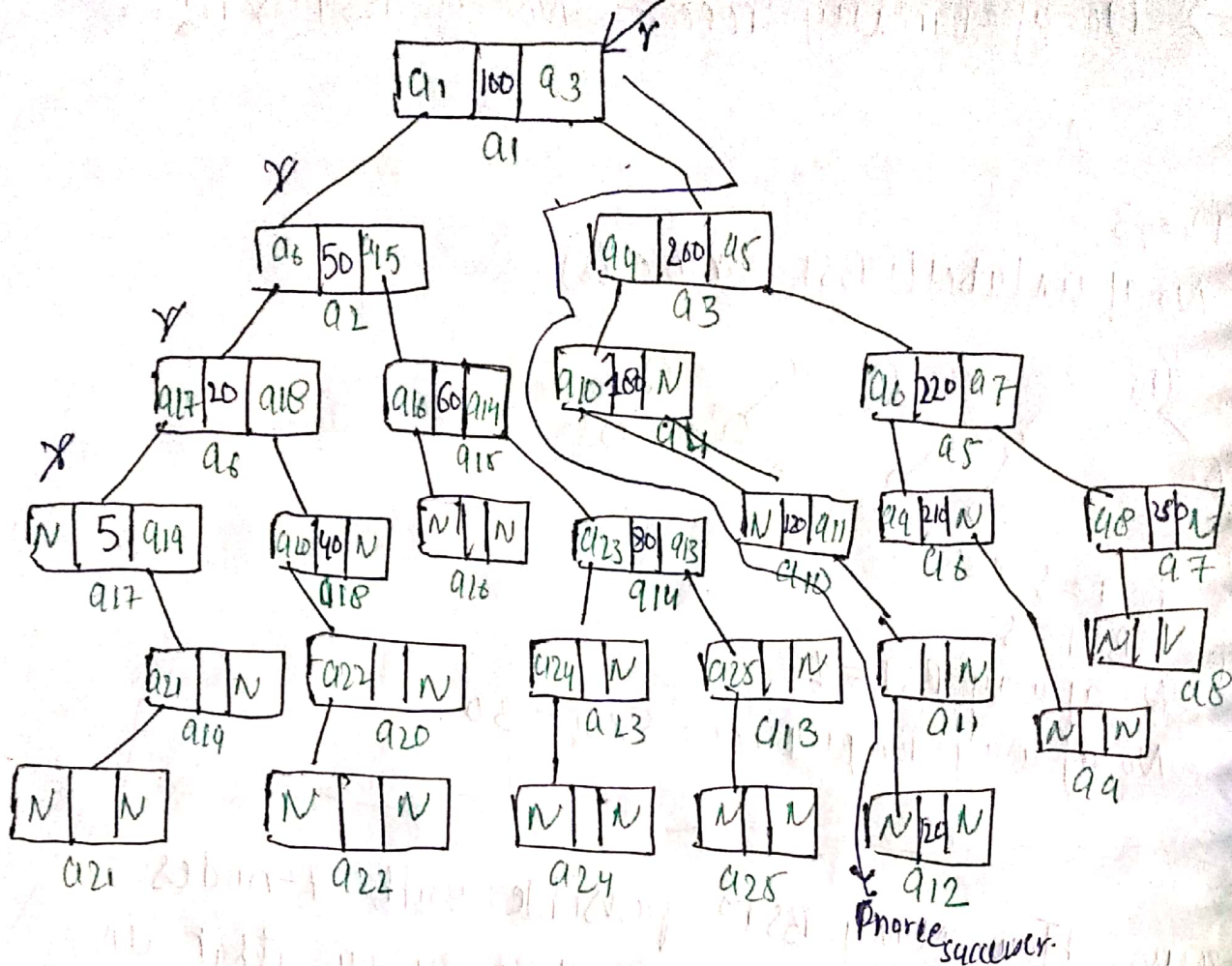
No of way BST = 2

No of way Binary Tree =  $5 \times 3! = 30$ .

Ques: How many BST's possible with 7-nodes.  
 $\{1, 2, 3, \dots, 7\}$  in such a way that in  
 all those BST's height will be 6



$$= 2^6 = \boxed{64}$$



ans: How much time it will take to find minimum element.

TC - find-min

|          | BC                                                     | WC                         | AC                 |
|----------|--------------------------------------------------------|----------------------------|--------------------|
| BST      | $O(1)$ left side has element                           | $O(n)$ all are at one side | $O(\log n)$ middle |
| AVL      | $O(\log n)$ Balanced no element left side not possible | $O(\log n)$                | $O(\log n)$        |
| BT       | $O(n)$ Min can be everywhere                           | $O(n)$                     | $O(n)$             |
| Max Heap | $O(n)$ you have to search all the nodes                | $O(n)$                     | $O(n)$             |

Comparison of nearly sorted Most of it contain that type of behaviour.

almost equal to unsorted array

coz it is meant for heap.

Pblm: How much time it will take to <sup>find</sup> element  $x$   
 Time Complexity:

|          | BC     | WC          | AC                                        |
|----------|--------|-------------|-------------------------------------------|
| BST      | $O(1)$ | $O(n)$      | $O(\log n)$                               |
| AVL      | $O(1)$ | $O(\log n)$ | $O(\log n)$                               |
| BT       | $O(1)$ | $O(n)$      | $O(n)$                                    |
| Max Heap | $O(1)$ | $O(n)$      | $O(n)$ - <small>check every path.</small> |

Pblm: How much it will take to ~~find~~ find element  $x$  is not existed.

|          | BC                               | WC          | AC          |
|----------|----------------------------------|-------------|-------------|
| BST      | $O(n)$ <small>Rj is null</small> | $O(n)$      | $O(\log n)$ |
| AVL      | $O(\log n)$                      | $O(\log n)$ | $O(\log n)$ |
| BT       | $O(n)$                           | $O(n)$      | $O(n)$      |
| Max Heap | $O(1)$                           | $O(n)$      | $O(n)$      |

→ root = 100  
Search = 120

↳ root = 500 (Max)  
Search = 1000  
Not there.

Insertion Element in BST:

Insertion Algo:

- ① Create node  $-x \Rightarrow O(1)$
- ② Find correct position of  $x \Rightarrow O(n)$  [WC]
- ③ Connect both  $\Rightarrow O(1)$

$O(n)$  [WC]  
 $O(1)$  [BC]  
 $O(\log n)$  [AC]

To Create BST for  $n$  nodes. Time complexity is  $O(n^2)$  worst case.

$O(n^2) \Rightarrow$  worst case

$O(n \log n) =$  Best Case & Avg. Case.

## Deletion

In Min-Heap tree inserting element will take.

$$BC = O(1)$$

$$WC \& A.C = O(\log n)$$

In Min-Heap tree you have to insert element.  $n$ . if  $n$  not exist.

$$T.C = n + \log n \\ = O(n)$$

InOrder: 5, 10, 15, 20, 25, 28, 40, 50, 55, 60, 63, 65, 80, 85, 90,  
95, 100, 120, 130, 150, 180, 200, 260, 280, 295, 290, 250.

Inorder  
Predecessor

Find the element  $x$ . Go 2-time left. & then go to highest right. <sup>with null</sup> called predecessor. Similarly successor.

## Deletion Algorithm

- ① Find - Node -  $x$
- ② (i) 0 - child  $\Rightarrow$  simply delete by adjusting few lines  $\Rightarrow O(1)$
- (ii) 2 - child  $\Rightarrow$  Simply - Delete by connecting grand-father & grand child.  $\Rightarrow O(1)$

ciii) 2-children  $\Rightarrow$  Replace n- data by predecessor data & successor data & delete predecessor & successor

$$a+b = O(n) [WC]$$

$$a+b = O(1) [BC]$$

$$a+b = O(\log n) [AC]$$

10/Sept/2014

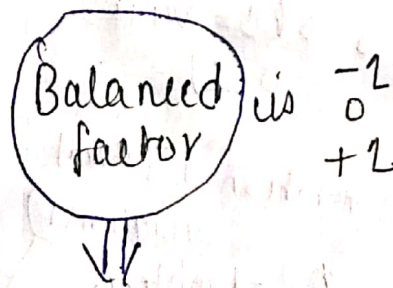
## AVL-Tree <sup>or</sup> (Height Balanced BST)

$$\begin{matrix} \text{Min} = \log n \\ \text{Max} = n \end{matrix}$$

$\log n$  - Always.

Ques: When can I say a BST is a Height Balanced BST  
 At every <sup>node</sup> balancing factor is  $-1, 0, +1$  then it can say that it is AVL Tree.

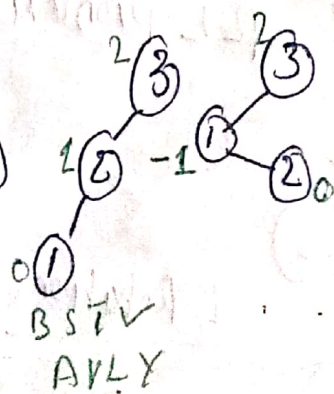
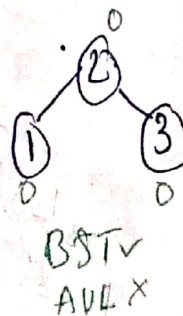
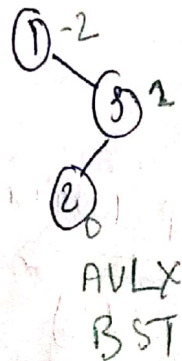
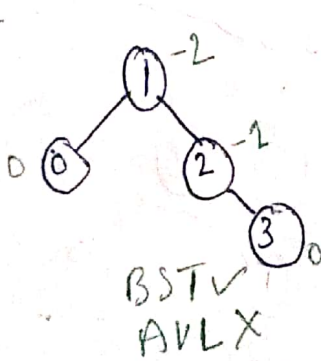
In BST  
 At Every Node



$$H(LST) - H(RST)$$

Ex 6

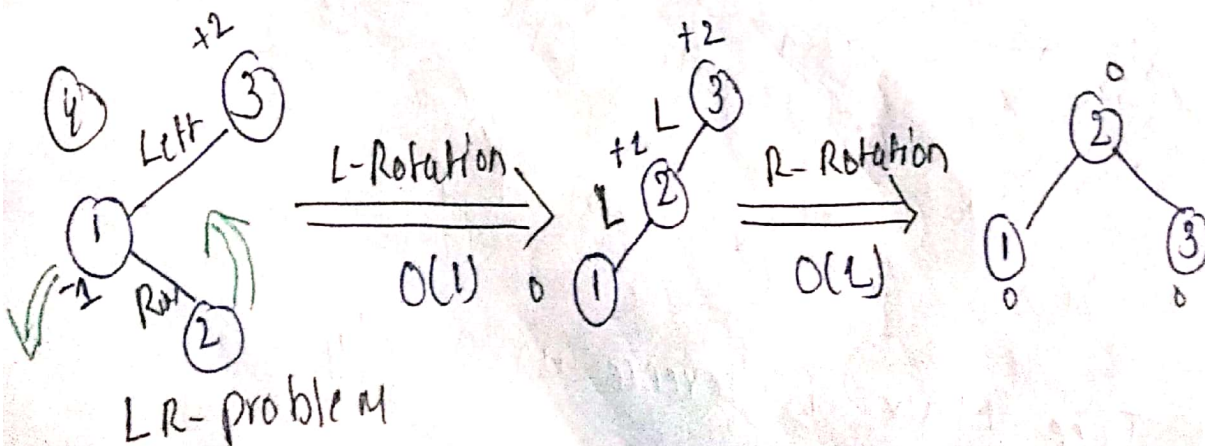
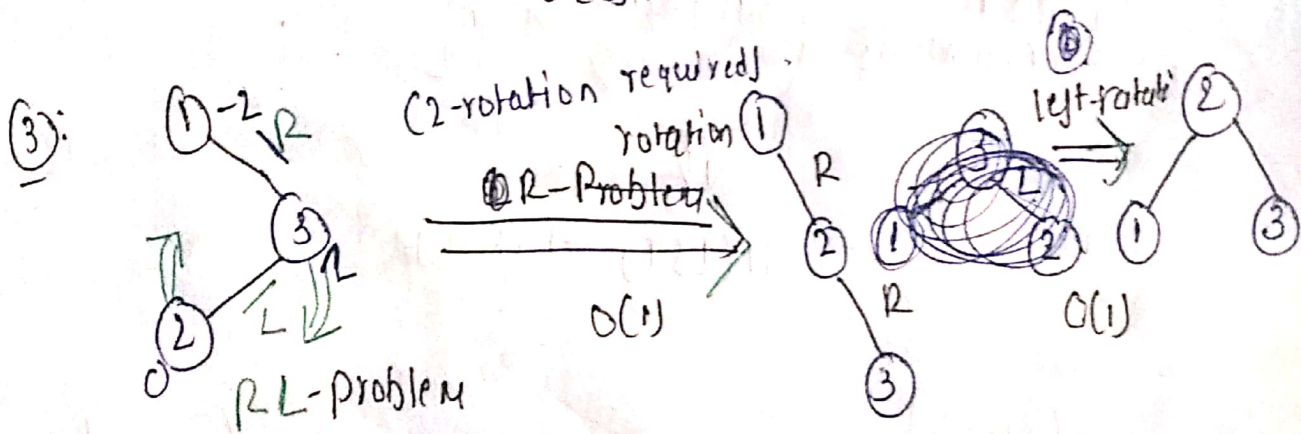
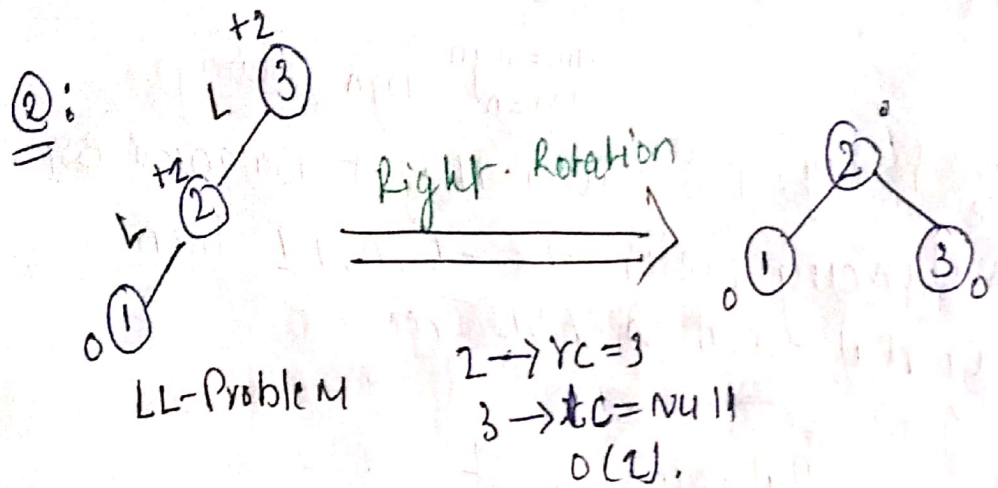
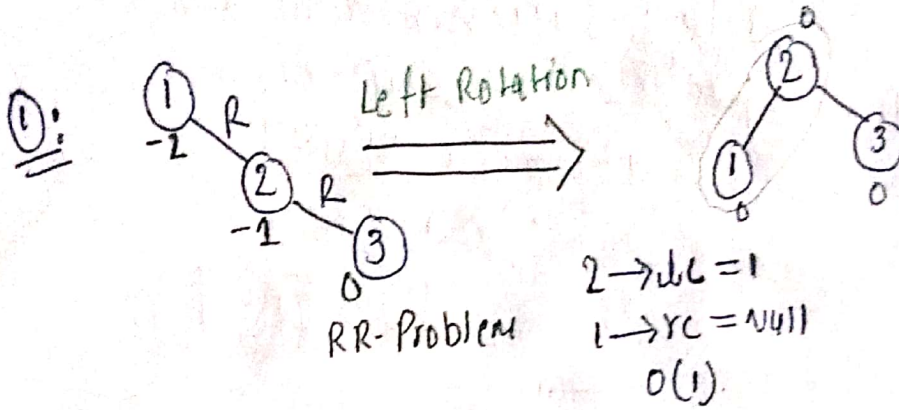
$n = (1, 2, 3)$



BST ✓  
 AVL X

In AVL Tree Max. Difference is 2 only  
 only 4 rotation they  
 [LL, RR, LR, RL]

# Rotations:



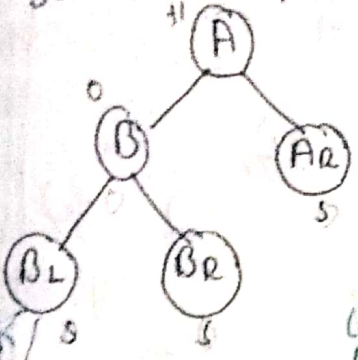
# Insertion in AVL Tree:

In AVL BST Insertion should be done at NULL place.

## LL-Problem

S = Height

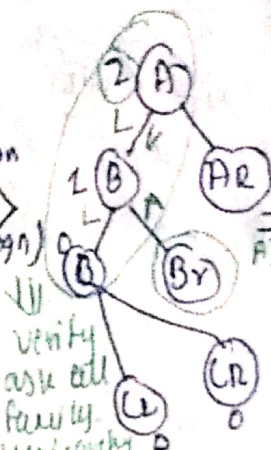
Insert = S+1



Insert - ~~Rotation~~  
 $O(1) + O(\log n) + O(1) + O(\log n)$

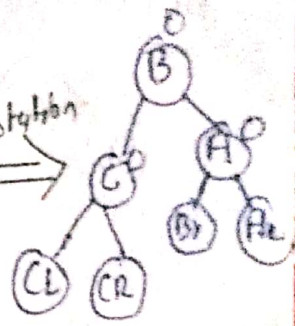
for creating node

finding the correct place  
 Insertion should be done at NULL place.



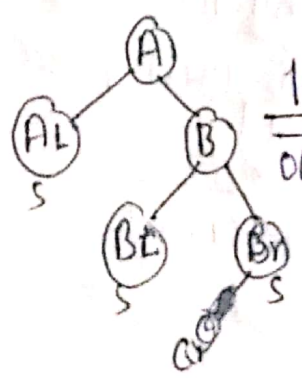
Right-Rotation

$A > Br > B$   
 $O(1)$

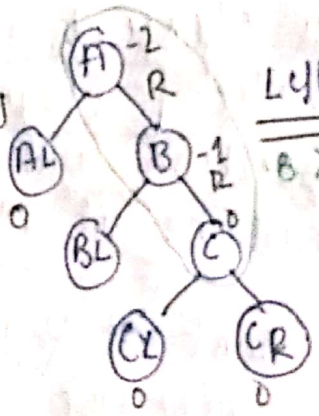


finding the correct place  
 why verify after all height of root path

## RR-Problem

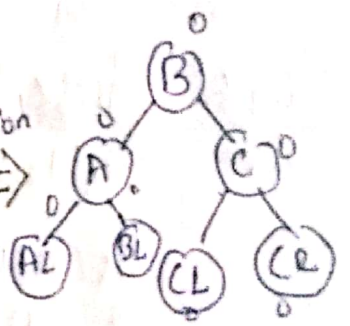


Insert - Br (S+1)  
 $O(1) + O(\log n) + O(1) + O(\log n)$

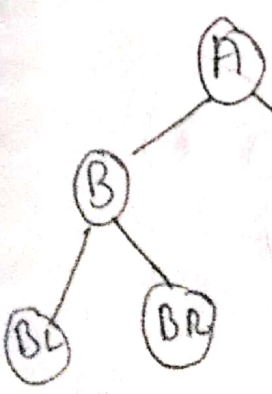


Left-Rotation

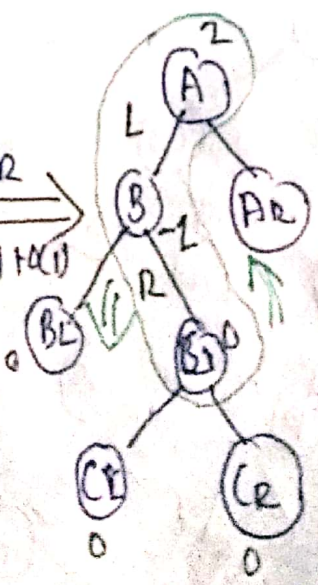
$B > BL > A$   
 $O(1)$



## LR-Problem:

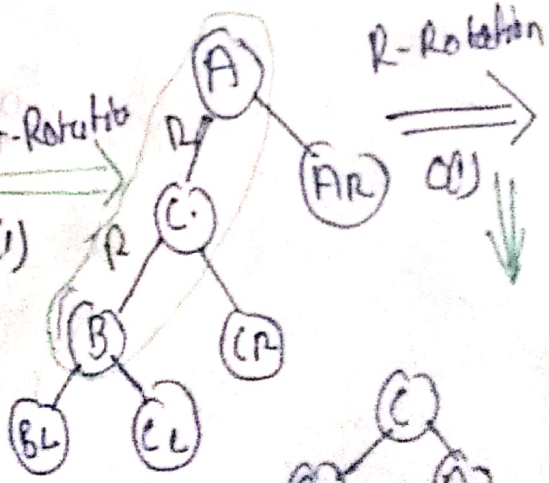


Insert - Br  
 $O(1) + O(\log n) + O(1) + O(\log n)$



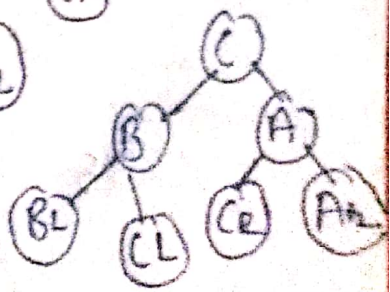
Left-Rotation

$O(1)$

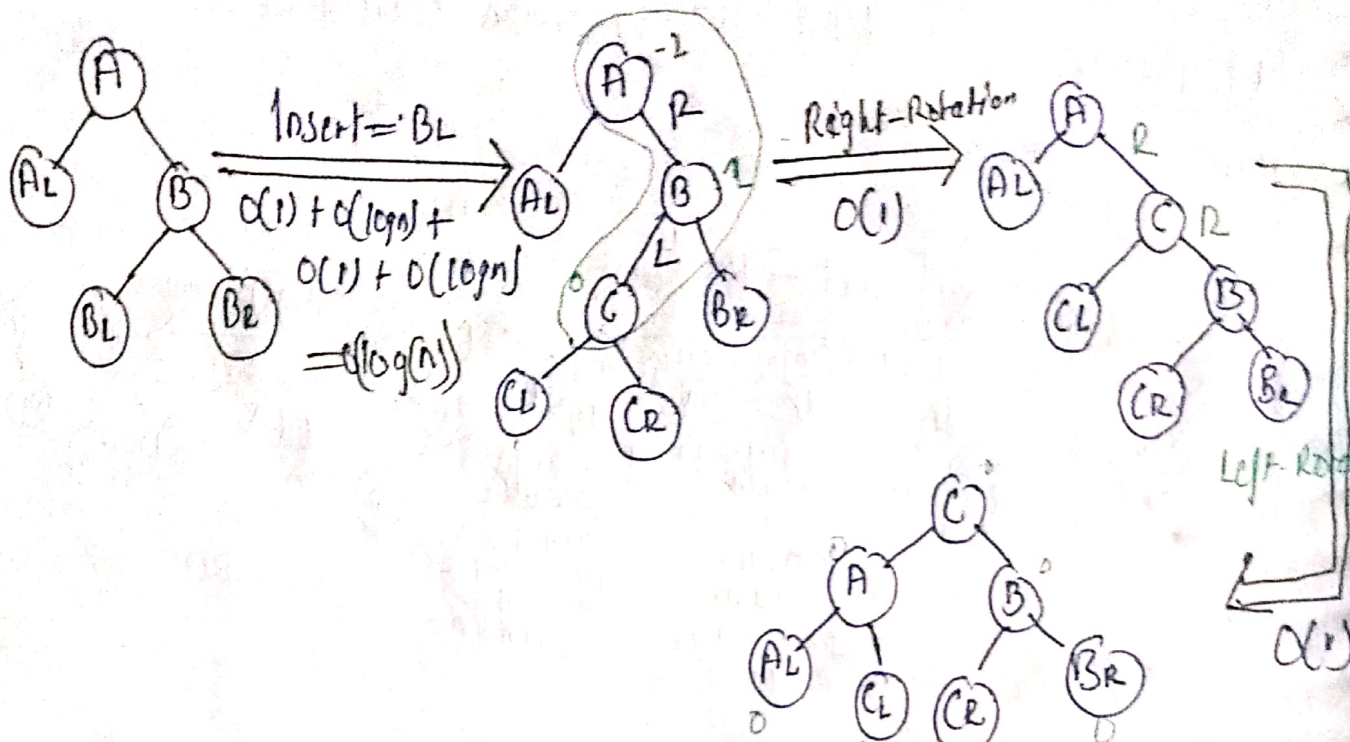


R-Rotation

$O(1)$



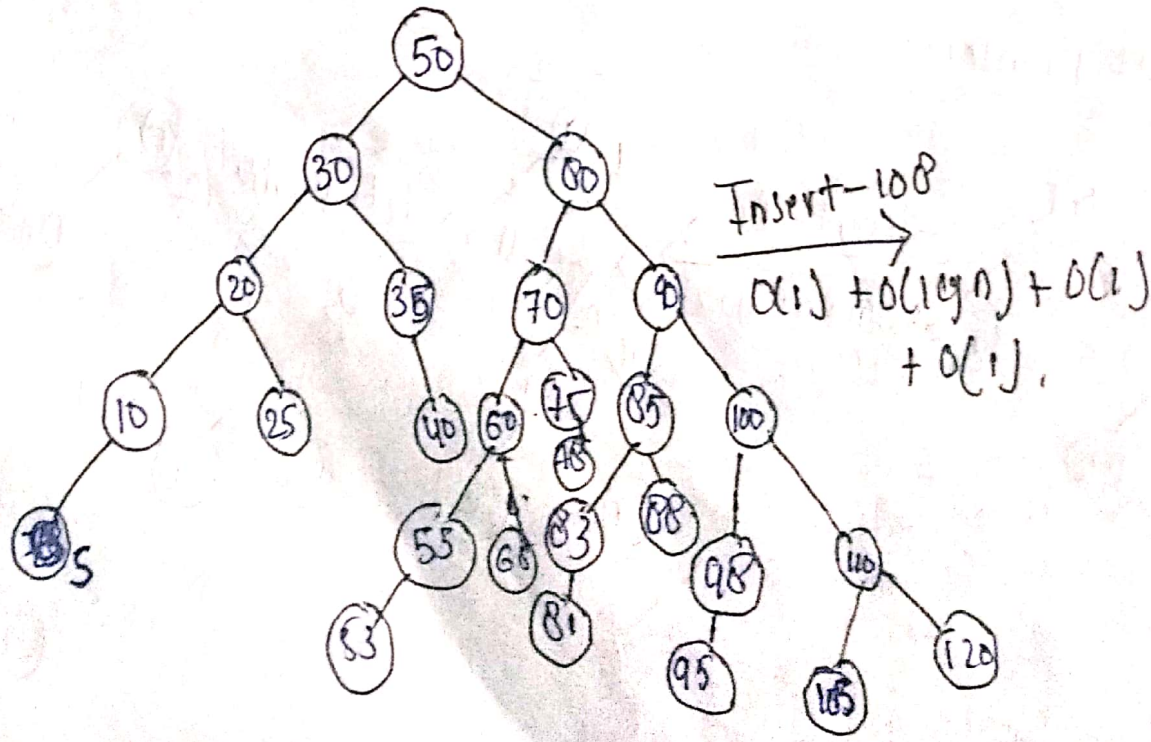
RL Problem:



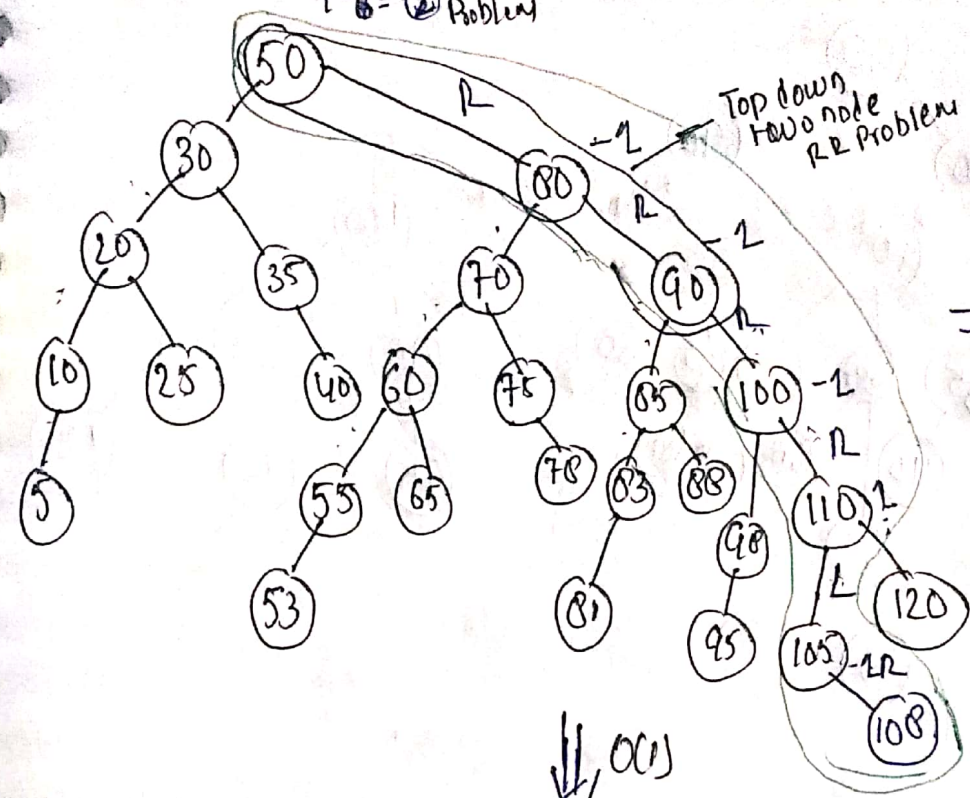
NOTE: Inserting a element into AVL Tree. will always take  $O(\log n)$  time. (why not  $O(1)$  because insertion will be done at null place & null will be <sup>here</sup> at the last level).

- In BST we dont do verify while inserting & deleting its element.

Ex 1



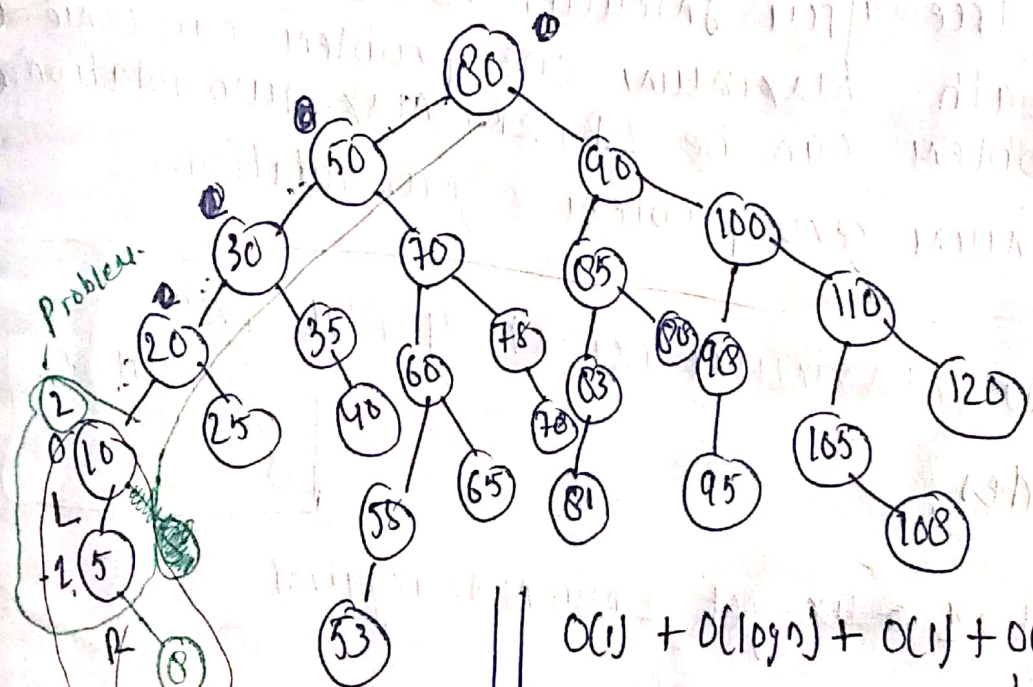
4-6 = 2 Problem



Top down  
Two node  
RR Problem

Left-rotation.

↓, O(1)



Insert element  
8

Problem.

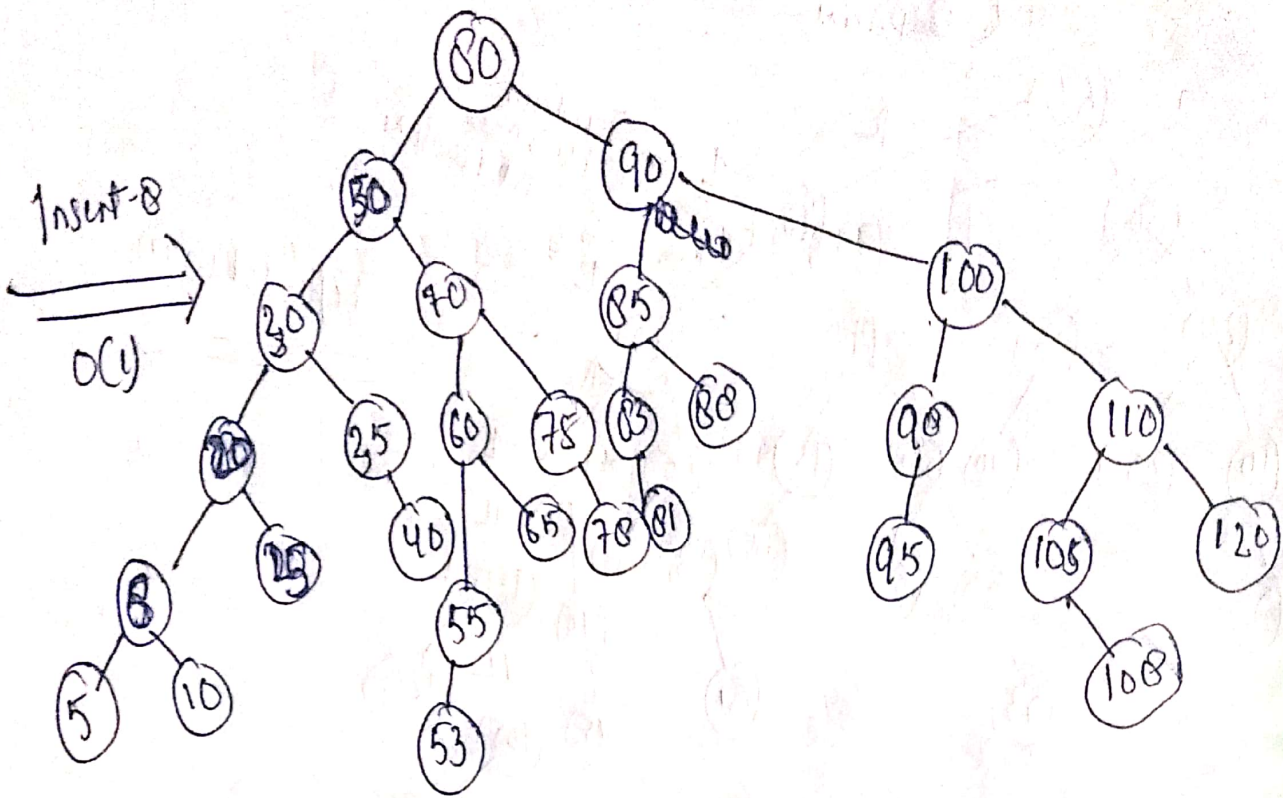


Left rotation

Left rotation.

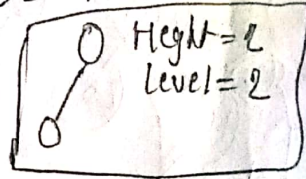
$O(1) + O(\log n) + O(1) + O(1)$

problem at starting  
So no need to verify further  
So  $O(1)$ .  
If you are not using  
Heap problem at the  
starting worst case  
case  $\log(n)$

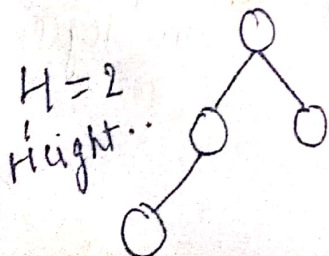
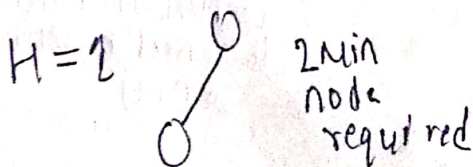


NOTE: In AVL Tree after insertion while backtracking is that path maximum. one problem can come. (that problem can be LR, RR, max. two rotation) & minimum zero problem. & zero rotation.

Ques: What is the maximum height of AVL Tree with 20 Nodes?



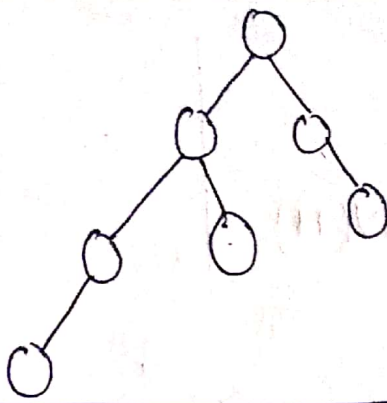
To get Height 0 ~~we get~~ 2 min node required



4 min node required.

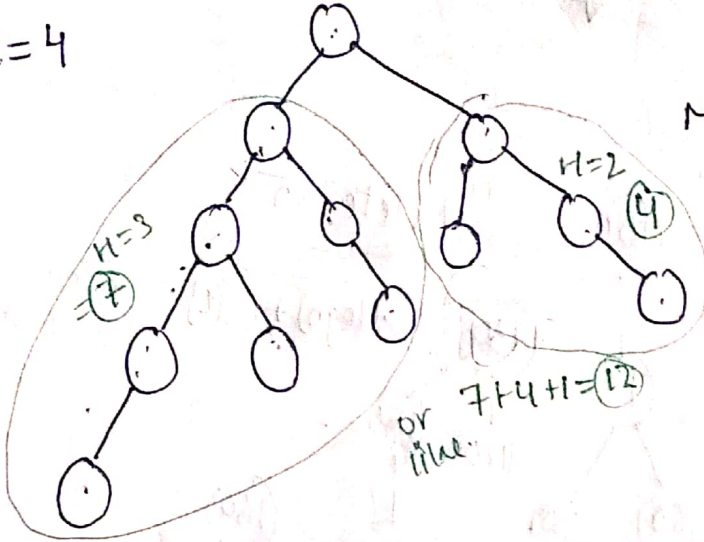
Ques: With the 4 Node what is the height of the AVL Tree  
or  
with H=2 How many node required?

H=3



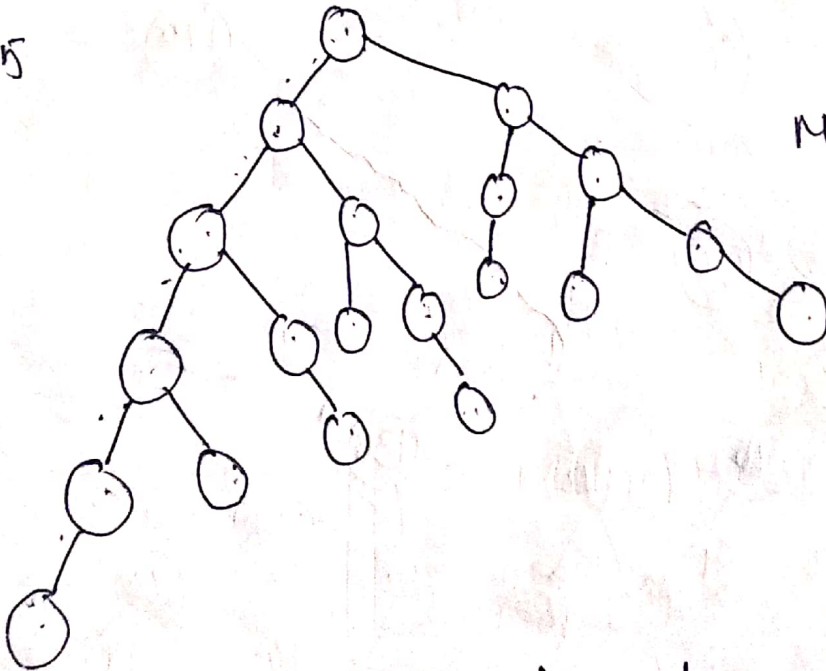
Min- 7 Nodes required

H=4



Min - 12 Nodes required

H=5



Min - 20 Nodes required

Or we can do by using formula .

MNN =  $\frac{\text{Min No of Node in the Height of AVL Tree .}}{\text{Min No of Node in the Height of AVL Tree .}}$

$$\boxed{MNN(6) = MNN(5) + MNN(4) + 2}$$

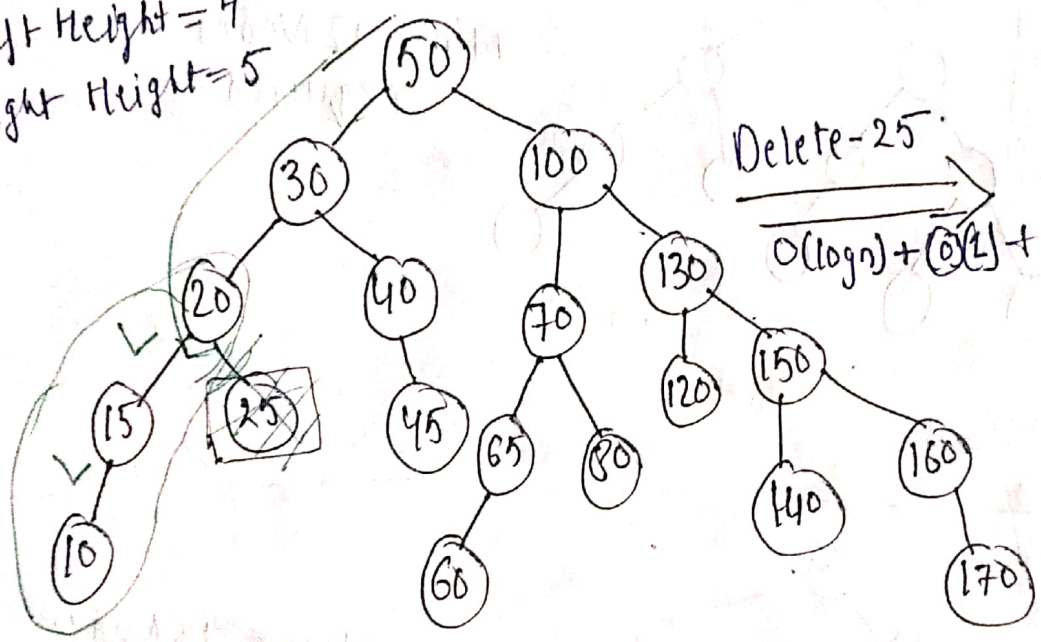
$$20 + 12$$

$$= 33$$

if  $H=1$  Nodes = 2  
 if  $H=2$  Nodes = 4  
 $M(N)(H-1) + M(N)(H-2) + 1 = M(N)(H)$

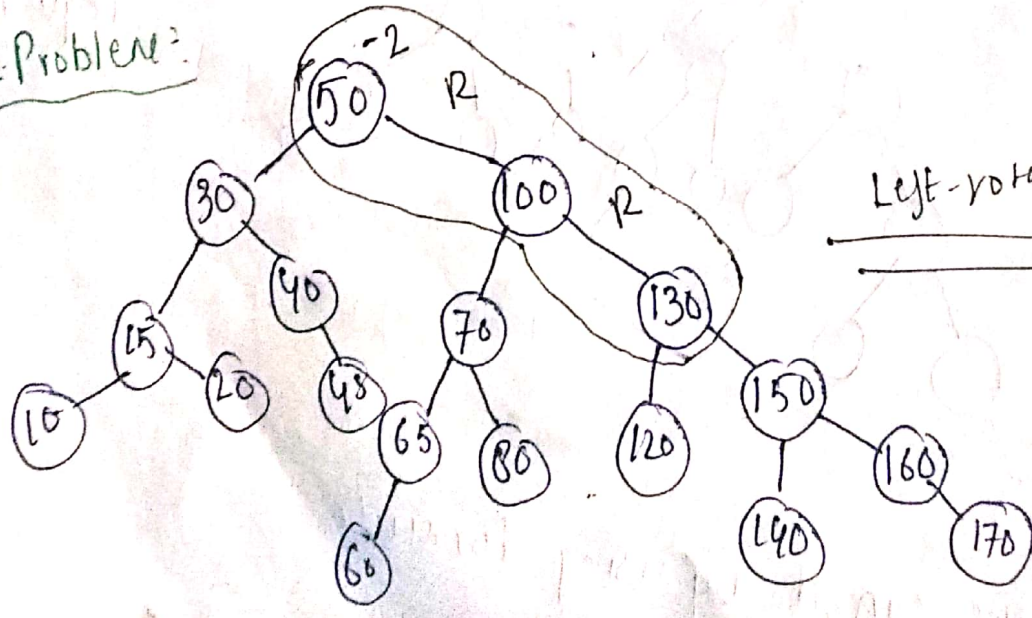
Consider the following AVL-Tree:

Left Height = 4  
 Right Height = 5



Delete-25  
 $O(\log n) + O(1) + O(\log n)$   
 verification

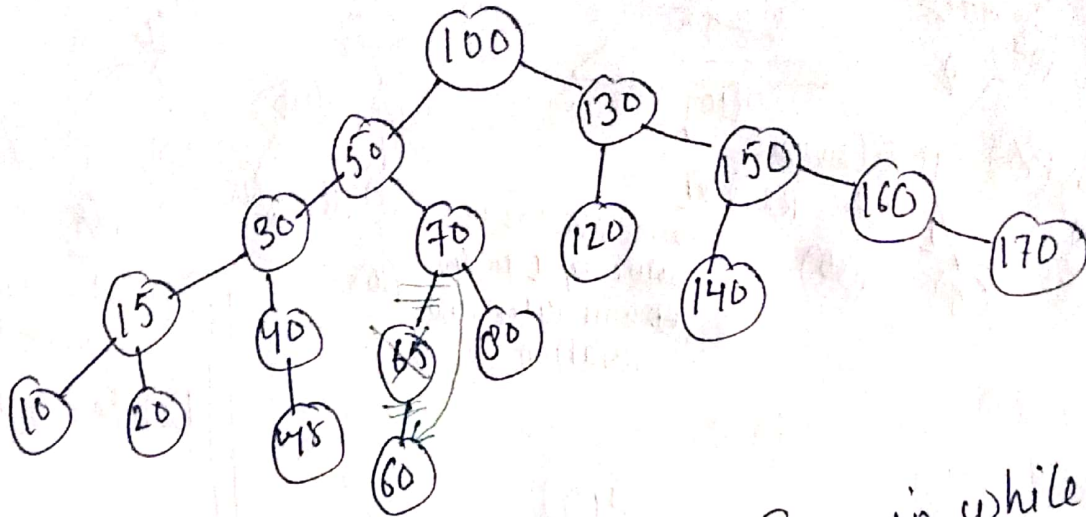
LL-Problem?



Left-rotation

If you delete element height will be increase or same  
 but never decrease  
 Now L Height = 3  
 R Height = 5  
 $= 5 - 3 = 2$  Problem

After deleting a node you need to verify/balck. problem while coming

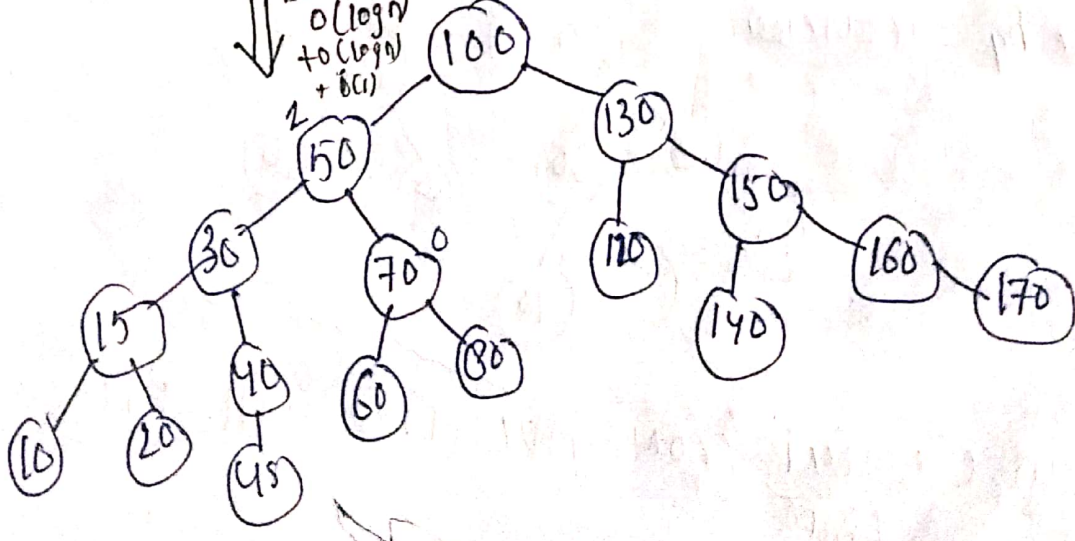


NOTE: In AVL Tree after deletion in while backtracking in that path maximum (logn) problems will come.

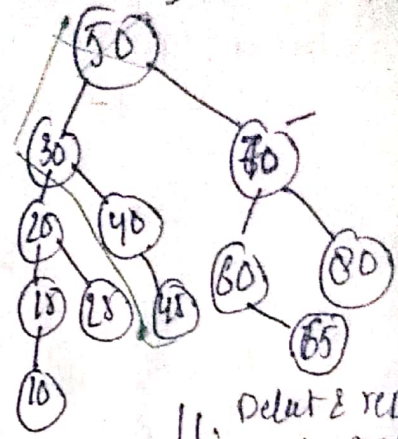
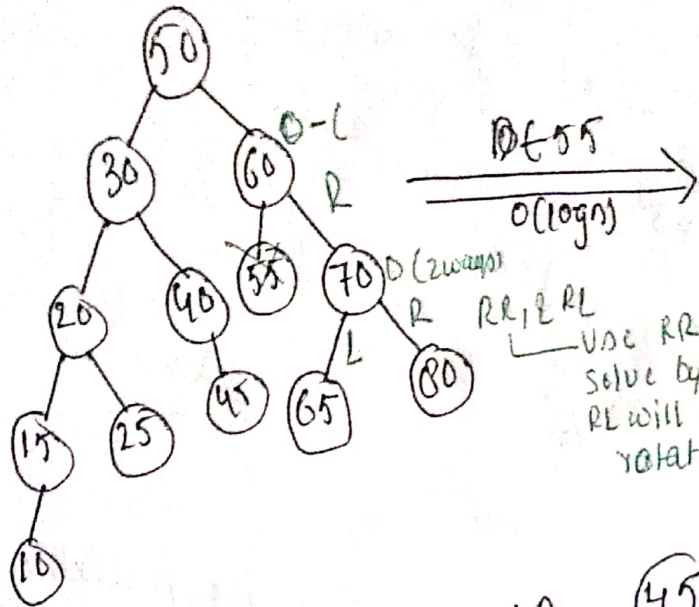
Delete - 65 in above Tree:

Simply Delete connect grandfather to son.

Delete = 65  
 $O(\log n)$   
 $+ O(\log n)$   
 $+ O(1)$



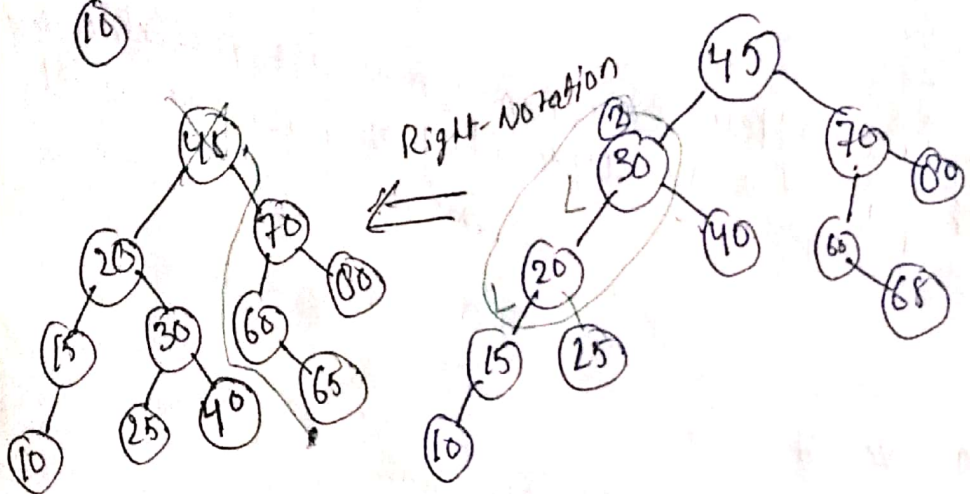
4-3 = 1



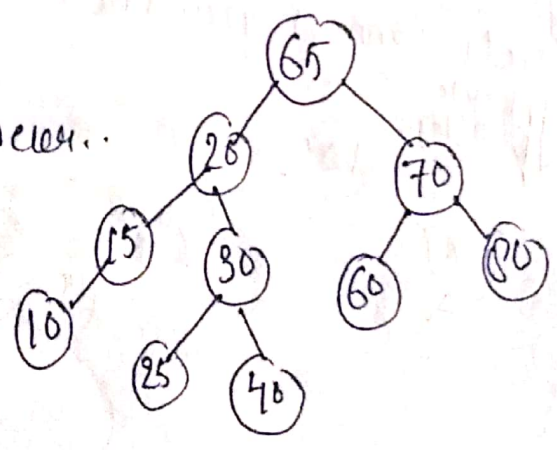
Delete & replace by predecessor

Delete - 50

Predecessor  $O(\log n)$  to find predecessor of 0-child.



Delete - 45 & replace by Successor.



Deleting an element from AVL Tree will take always  $\log n$  time

Q. In which one of the following data structure following two operation <sup>can be done</sup> will take order of  $O(\log n)$  time  
 ① Insertion      ② Deletion.

a) AVL

b) Min-Heap. → Deletion → Max-Log(n)  
Min-0(1)

c) Both

d) None-of-these

Ques: If the above Question condition is

① Insert element n is not there.

② Deletion

a) AVL

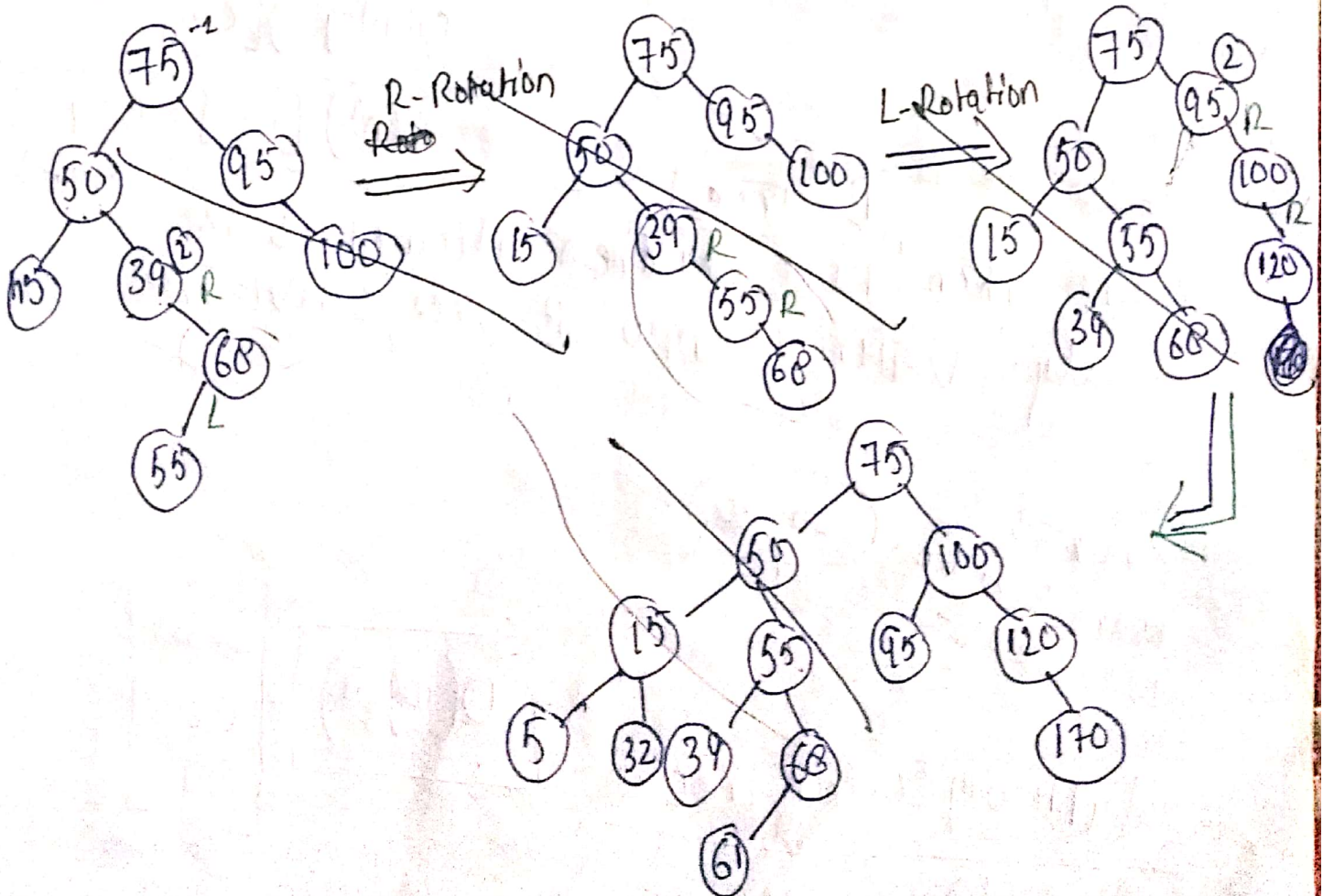
b) Min-Heap - for insert element if which is not there it will take O(n) time to scan whole tree.

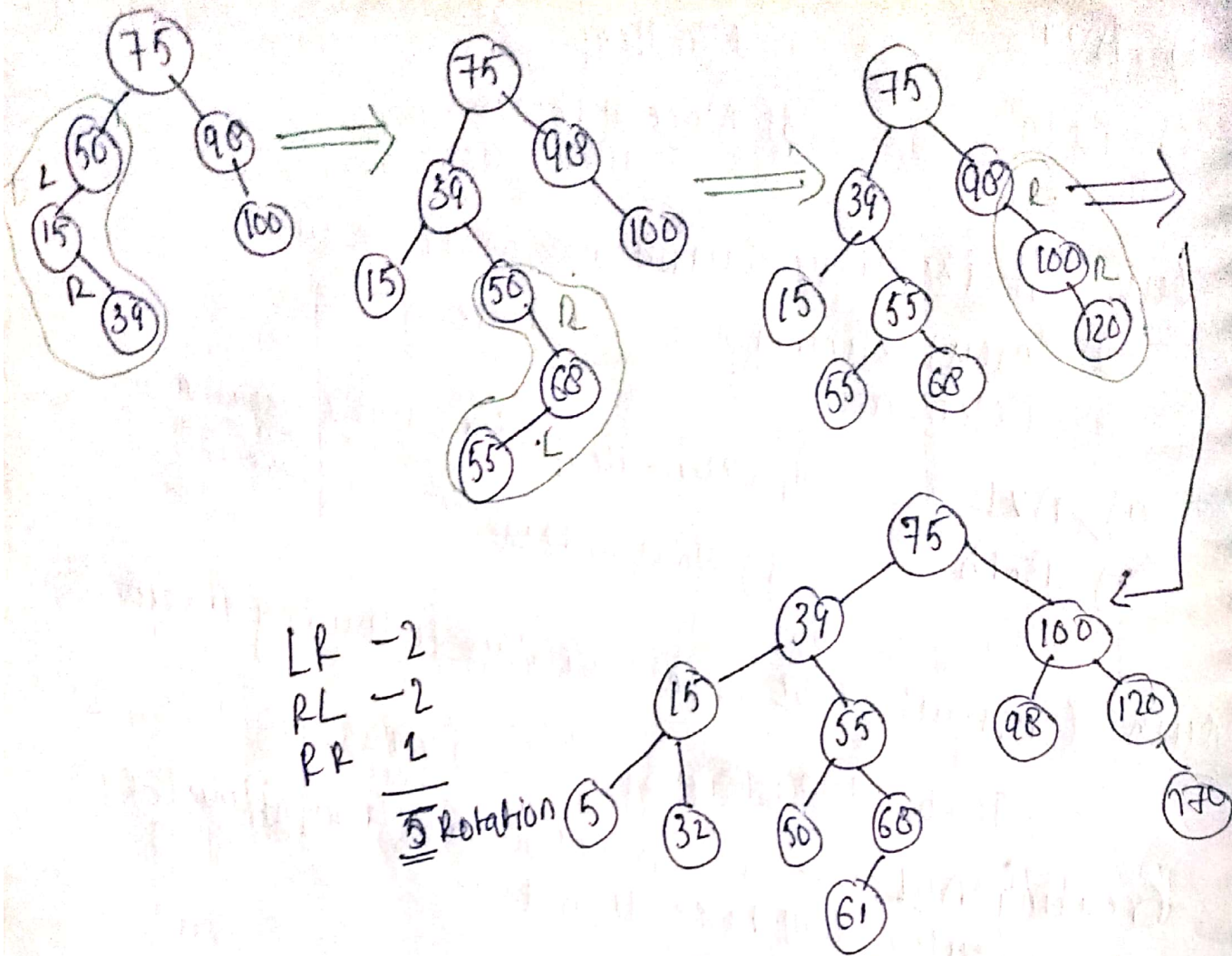
c) Both

Ques: Construct AVL Tree. for the following elements:

75, 98, 50, 100, 15, 39, 68, 55, 120, 170, 5, 32, 61

Creating AVL Tree for n-elements,  $(n \log n)$  time [EC]  
O(2) insertion will take  $\log n$  time.





LR -2  
 RL -2  
 RR 1

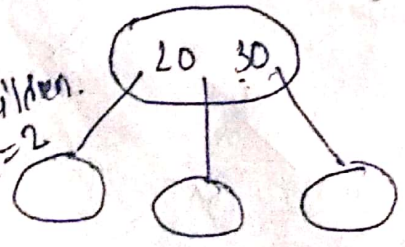
5 Rotation

$$\frac{O(n \log n) [EC] = AVL}{O(n^2) [wc] = BST.}$$

B-Tree

More than two children allowed is the only difference b/w B-Tree & AVL Tree.

Order = 3  
 Max = 3 children.  
 Max = data = 2



Multiway Search Tree.

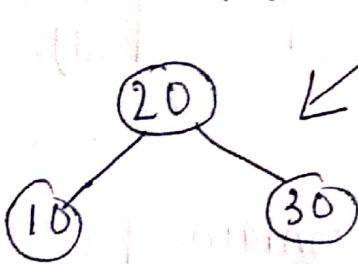
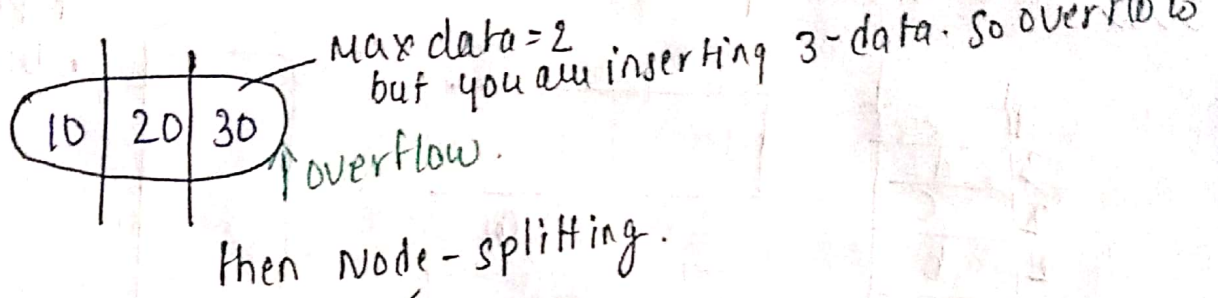
Balanced Tree.

$O(\log_3 n)$

I.S  
 D.S  
 S.T  
 Searching

Here No Rotation.

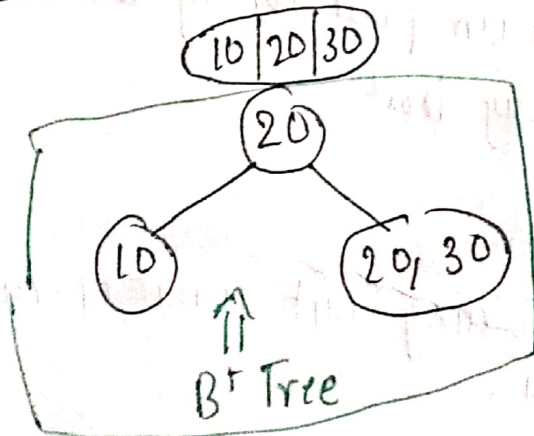
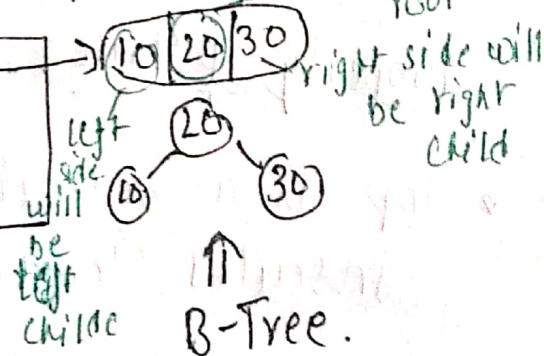
Max = data = 2



In B tree - Inserting time. overflow problem arise  
Deleting data underflow problem there

Sol<sup>n</sup>

If overflow: Do Node-Splitting  
If underflow: Do Node Combining



← Whenever you are Splitting like this..

19/Sept/2019

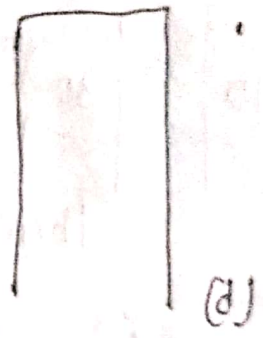
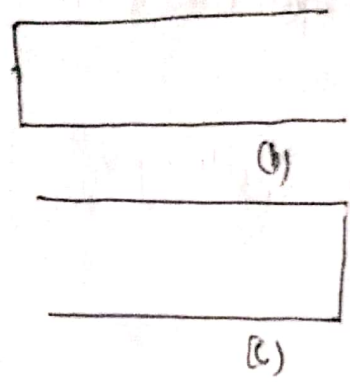
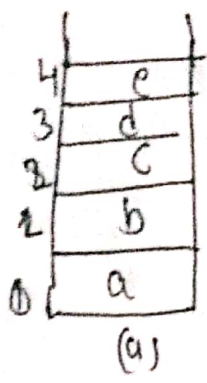
# Stack

It is a linear data structure, in which element can be inserted at one end.

Def:

One side open, another side closed.

Representation  
• Diff in Different  
Books



Top = 4

↳ Top is a variable which contains position of Top most element.

Property of Stack: LIFO (or) FILO

- Top is a variable which contains position of newly inserted element - In Queue we say rear.
- Top is a variable which contains position of an element deleted - In Queue we say front.

## ADT of Stack.

ADT: Abstract Data Type. It is nothing but what operation you can perform with them..

ADT of stack



Push()

Pop()

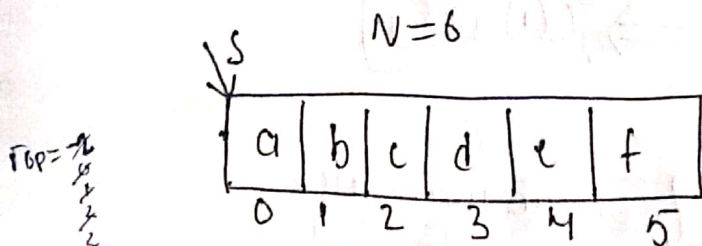
Implementing Stack: Nothing but Push & Pop operation writing with which data structure you are using like, array, array, linked list etc.

```

Push() Pop()
{ {
} }

```

Implementing Stack using array:



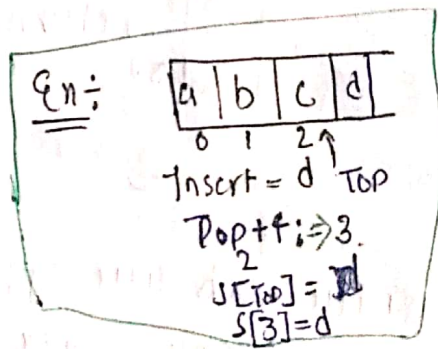
Push: Op<sup>n</sup>

Push(int x)

```

{
 if (Top + 1 == N)
 {
 Pf("stack overflow")
 }
 else
 {
 Top++
 S[Top] = x
 }
}

```



⇒ O(1) [EC]

[First inc. then store. if we store without inc. then ~~data~~ overwriting]

= also write

S[Top++] = x ✗

S[++Top] = x ✓

Pop Op<sup>n</sup>:

~~POP(int y)~~

~~int y~~

POP()

```

int y
if(Top == -1)

```

```

{
 pf("stack is underflow")
 exit();
}

```

⇒ O(1) [EC]

else

```

{
 y = s[Top]
 Top--
 return(y)
}

```

or can write also  $y = s[Top--]$

→ y  
y

- POP will return but push not.
- Push take variable element but POP didn't take.
- Push first incr. & then store
- POP first store & then Decrement.

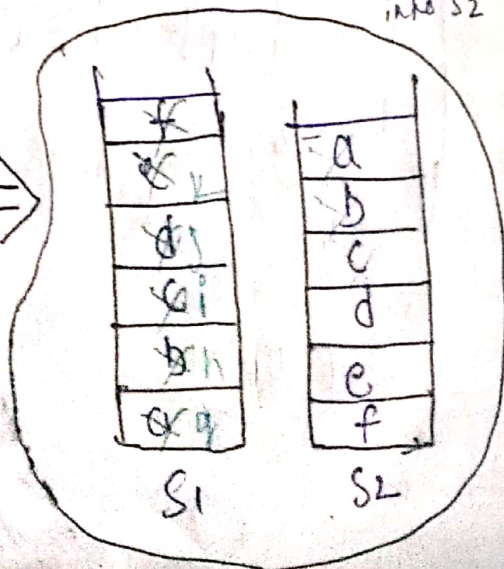
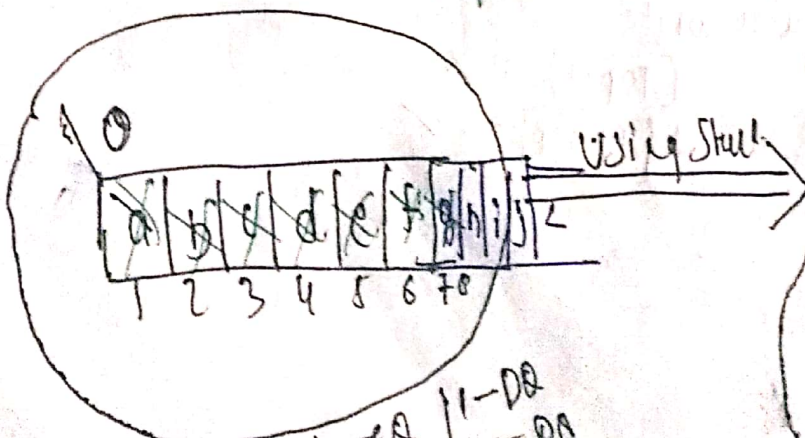
Stack & Queue both are opposite. so there will be chance  
 Implementing Stack using Queue. or Queue using Stack  
 in a question in gate.

### Implementing Queue Using Stack;

Enqueue()  
Dequeue()

Push()  
Pop()

Whenever Dequeue happen it will reverse all the element in 2nd stack. Shift all elements S1 into S2



1-EO | 1-PO | 1-EO | 1-DO  
 2-EO | 1-DO | 4-EO | 2-DO  
 3-EO | 1-DO | 1-DO

1-EO = 1-push-S1  
 2-EO = 2-push-S1  
 3-EO = 3-push-S1  


---

 1-DO = 6-pop-S1  
       = 6-push-S2  
       = 1-pop-S2

---

 1-DO = 1-pop-S2  


---

 1-DO = 1-pop-S2  


---

 1-EO = 1-push-S1  


---

 4-EO = 4-push-S1  


---

 1-DO = 1-pop-S2  


---

 2-DO = 2-pop-S2  


---

 1-DO = 5-pop-S1  
       = 5-pop-S2  
       = 1-pop-S2

$O(n)$   
[WC]

```

void EO(x)
{
 push(s, x)
}

```

Enqueue  $\Rightarrow O(1)$

```

int DO()
{
 if (s2 is not empty)
 return (pop(s2))
 else
 {
 while (s1 is not empty)
 {
 x = pop(s1)
 push(s2, x)
 }
 return (pop(s2))
 }
}

```

Dequeue  $O(1)$  [AC BC]  $\Rightarrow O(n)$  [WC]

If the class is like this: Make Dequeue cheaper.

Queue  $\Rightarrow$  Enqueue / Dequeue

Stack = push() / pop() / Reverse()

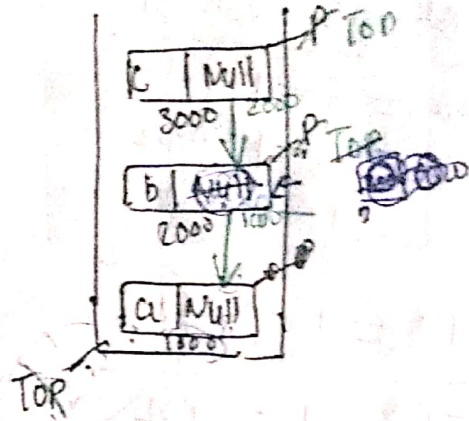
It will reverse within stack

EO  
 ↓↓  
 Reverse  
 push  
 Reverse

DO  
 ↓↓  
 pop()

# Implementing Stack Using Linked List:

Push(n)



Push(n)

```

{
P = malloc(C)
if (P == NULL) {
 printf("Stack Overflow");
 exit(1);
}
else {
 P->next = TOP;
 TOP = P;
}
}

```

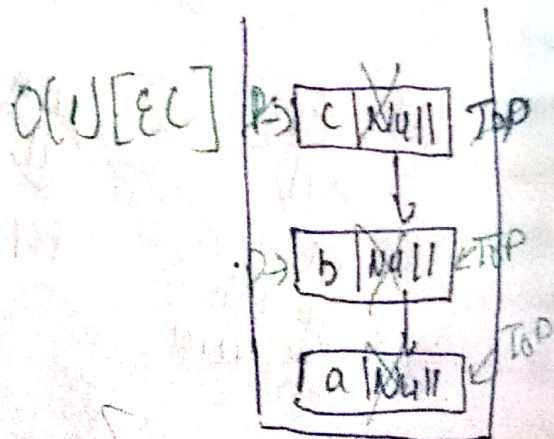
Pop()

Pop()

```

if (TOP == NULL) {
 printf("String Underflow");
}
else {
 y = TOP -> data;
 P = TOP;
 TOP -> next;
}

```

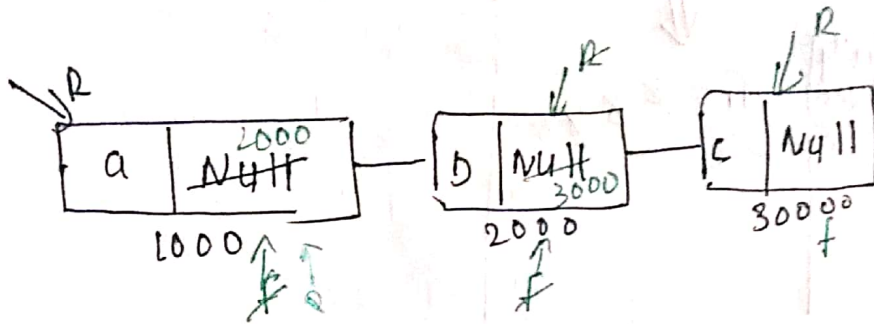


```

free (P)
P = Null
return (y)
}

```

Queue Using linked list:



EnQueue:

```

EOC()
{
P = malloc()
if (P == Null) {
 printf("Queue Overflow");
}
else.
R -> next = P
R = P
}

```

⇒ O(1)

DeQueue:

```

DOU ⇒ O(1)
{
if (F == Null)
 print("Queue Empty");
}
else
P = F
F = F -> next
y = P -> data.
Free(P)
P = Null
return(y)
}

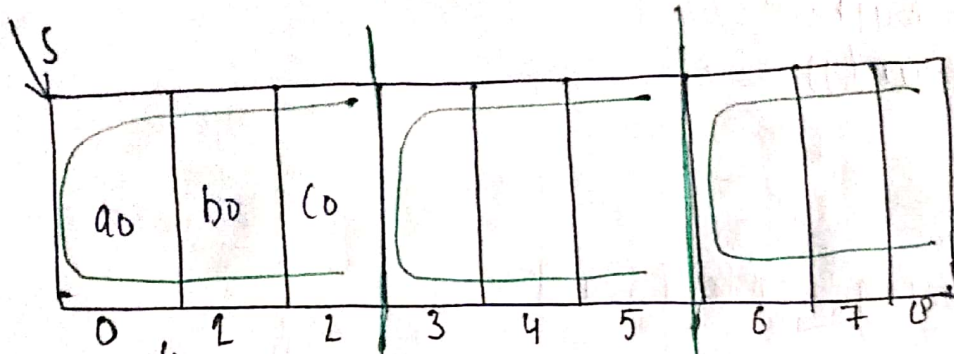
```

Hw: Implement Stack using Queue:

Implementing Multiple Stack in single array:

↳ Advantage: You can execute multiple program in a single array like, gcd, Fibonacci, LL, etc. (recursive program)

$N = 9$  (Size of array)  
 $M = 3$  (No of stacks)  
 $N/M = 9/3 = 3$  (Size of 1 stack)



(Stack # Top)  $T_0 = -1$  (Initially outside)  
 $\times 2$   
 Push()  
 $\epsilon$   
 $TOP + 1$   
 $S[T_0] = x$

$T_1 = 2, 3, 4, 5$

Push condition for overflow

When can I say Stack No. 55 is full?  
 Stack No. 55 Current Top == Stack No. 56. Initially Top.

$2^{nd}$  - stack = Initially Top of Stack.

$$2 \times 3 - 1 = 5$$

(0) 2 Before size of stack

$0^{th}$  - stack - ITOS

$$0 \times 3 - 1 = -1$$

$25^{th}$  - stack = ITOS.

$$25 \times 3 - 1 \Rightarrow 74$$

$i^{th}$  stack - ITOS

$$i \times \frac{N}{M} - 1$$

When can I say stack No 10 is empty.  
 stack No 10 of  $arr[0 \dots 9]$   $==$  Initially top of the stack POP.

Could Pop for underflow

```

 Push:
 void Push(x, i)
 {
 if (T_i == (i+1) * N / 2 - 1)
 {
 Pf("stack is overflow");
 exit(1);
 }
 else
 {
 T_i++;
 s[T_i] = x;
 }
 }

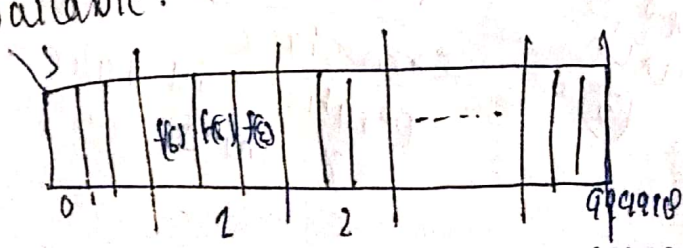
```

```

 Pop(i)
 {
 if (T_i == i * N / 2 - 1)
 {
 Pf("stack is underflow");
 exit(1);
 }
 else
 {
 y = s[T_i];
 T_i--;
 return (y);
 }
 }

```

Using above program we can implement multiple stack in single array but it is not efficient because if one stack is full all other remaining are empty ~~then~~ we will get error msg "stack overflow" ~~even even~~ even though space available.

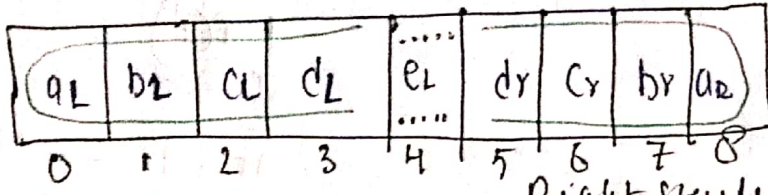


$N = 99999$   
 $M = 33333$

f() call  
 space available even though we are not able to place function.

# Implementing Multiple Stack in Single Array with Efficiency

$n = 9$  (Array size)



Left Stack

Right Stack

$TL = -1$  (initially outside)  
 $x, x, x, x, x$

$TR = 9$  (initially outside)  
 $x, x, x, x, x$

Push

$TL++$   
 $S[TL] = x$

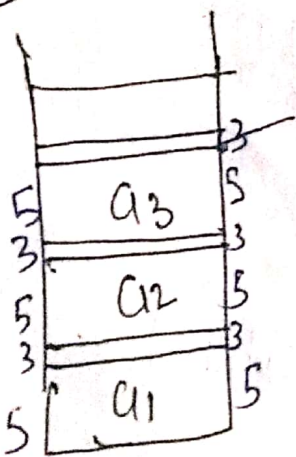
$TR--$   
 $S[TR] = x$

If this is a question, what is the overflow condition

- a)  $TL == N-1$
- b)  $TR == 0$
- c)  $TL == TR-1$
- d)  $TR == TL-1$

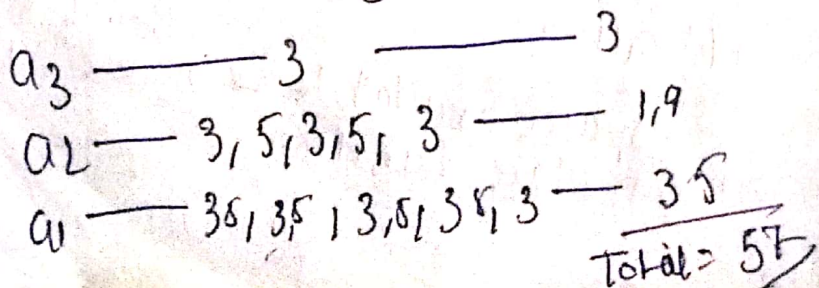
## Average Lifetime of an element in stack:

ex:  $n = 3 (a_1, a_2, a_3) \Rightarrow$  3 push con  
 follow  
 3 pop con



elapsed time  
 4-units  
 (3)

each stack operation will take  
 $n$ -units.  
 (5)





# Recursion

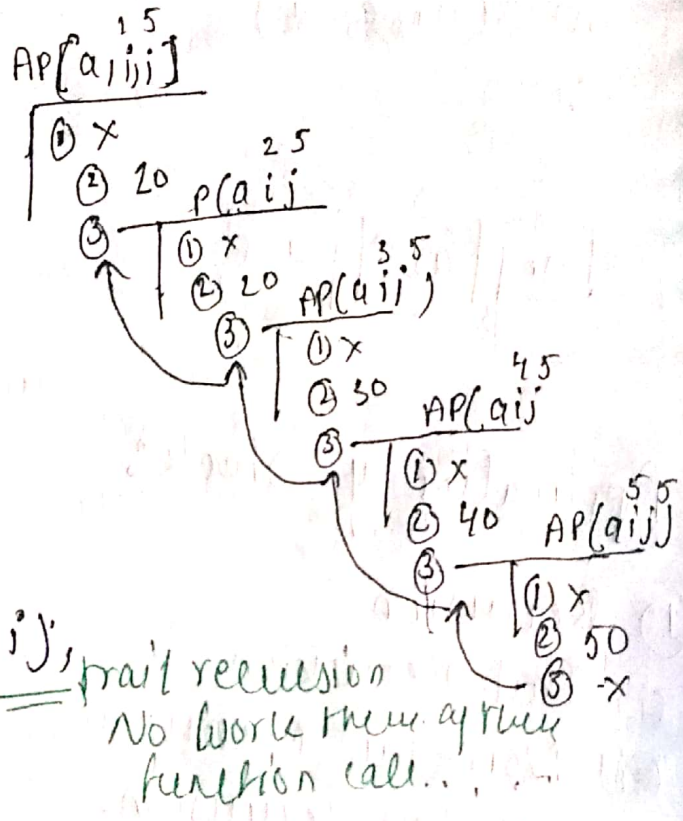
Write a C Program to print the given array of  $n$  elements.

Array printing (a, i, j)      Ex:  $A \begin{bmatrix} 10 & 20 & 30 & 40 & 50 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$

```

{
 if (i == j)
 {
 pf(a[i]);
 return;
 }
 else
 {
 pf(a[i]);
 Arrayprinting(a, i+1, j);
 }
}

```



Disadvantage: Taking more space

Tail - Recursion? → you are calling recursion func at last of line that no work there so misuse of stack space.

① In the given recursive program after the function call no work there. Then it is known as tail recursive program.

Example: Above Program.

Tail recursion taking wasting lot of stack space & time

③ The advantage with the tail-recursion is we can write non-recursive program easily with the help of while loop. Take all inside the while loop.

### Non-Tail Recursive Program:

Ex:

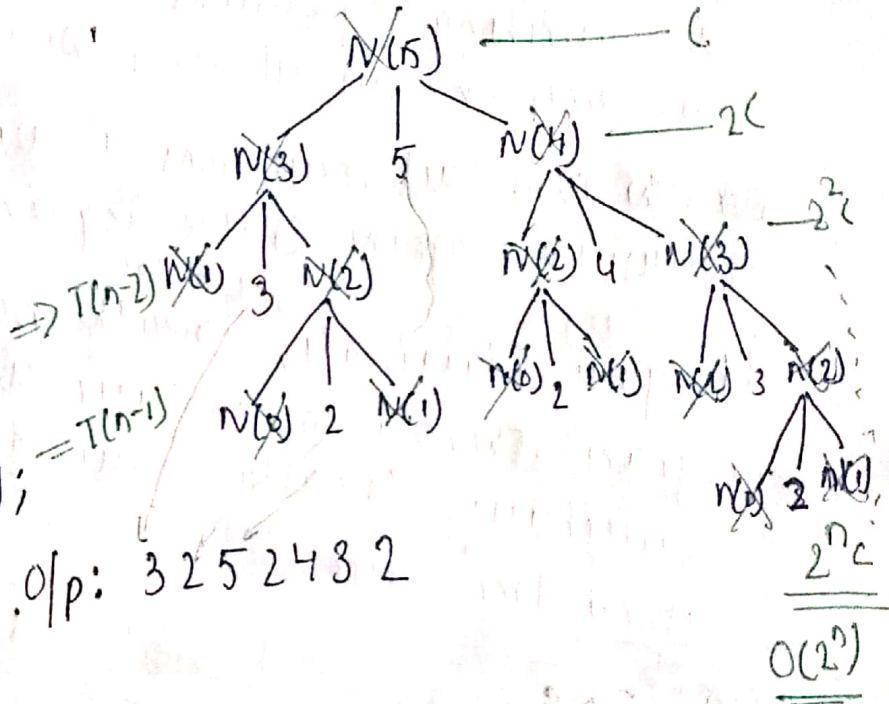
NTR(n)

i/p = n = 5

```

{
 if (n ≤ 1)
 return;
 else
 {
 NTR(n-2);
 P(n);
 NTR(n-1);
 }
}

```



~~Time Complexity:~~

All function call  
~~Without DP:~~

Time Complexity:  $2^n = O(2^n)$

Space: i/p + extra  
 $2B + nB$   
 $O(n)$

Distinct function call  
with DP:

Time Complexity:  $n = O(n)$

Space: i/p + extra.  
 $2B + nB$   
stack table  
 $2n = O(n)$

NOTE:

- ① In the given recursive program given the function call some work is there then it is known as non-tail recursive program. above problem is a example for that.
- ② It is not wasting the stack space.
- ③ The Difficulty with the non-tail recursive program is writing equivalent non-recursive program is complex but it is possible.

For the Non-tail recursions, if we write the non-recursive program stack required.

- ④ If you eliminate <sup>recursion</sup> from the non-tail recursive program then also we are using stack data structure.. (Really required you have to use it if not then same it).

```

A(5)
{
 Push(5)
 Push(3)
 Push(0)
 Pop(0)
 Push(2)
 :
 Pop(3)
 Pop(4)
 Pop(5)
}

```

Equivalent Non-Recursive Program for above problem. Here user will be written the push & pop operation. Here also stack required.

It required it is compulsory no-use it not no required.

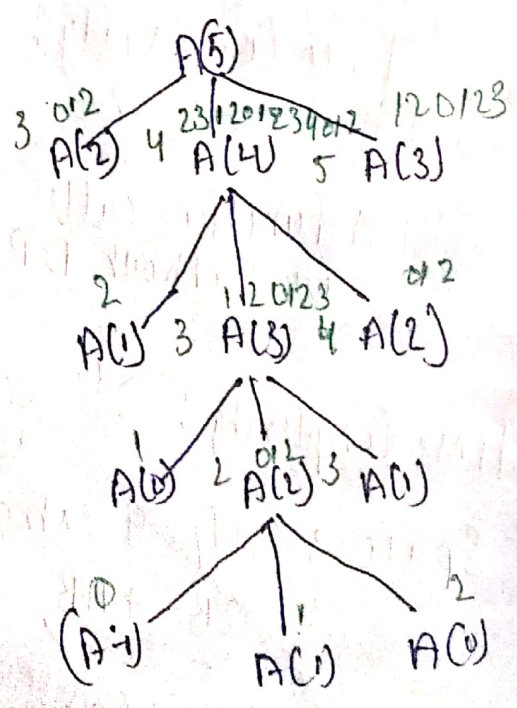
Ex ②:

```

A(n)
{
 if (n <= 1)
 return;
 else
 {
 Pf(n-2)
 A(n-3)
 Pf(n-1)
 A(n-1)
 Pf(n)
 A(n-2)
 }
}

```

i/p: ⑤ Solve:  $A(n-1)$   
 It cover all the possibility.



O/p: 3 0 1 2 4 2 3 1 2 0 1 2 3 4 0 1 2 5 1 2 0 1 2 3

Without DP

Space Comp:  $2B + nB$   
 $\boxed{O(n)}$

Time Compl:  $\boxed{O(3^n)}$

With DP

Space Complexity:  $2B + nB + nB$   
 $\boxed{O(n)}$  extra space table

Time Complexity:  $\boxed{O(n)}$  Distinct function calls?

INDIRECT RECURSION:

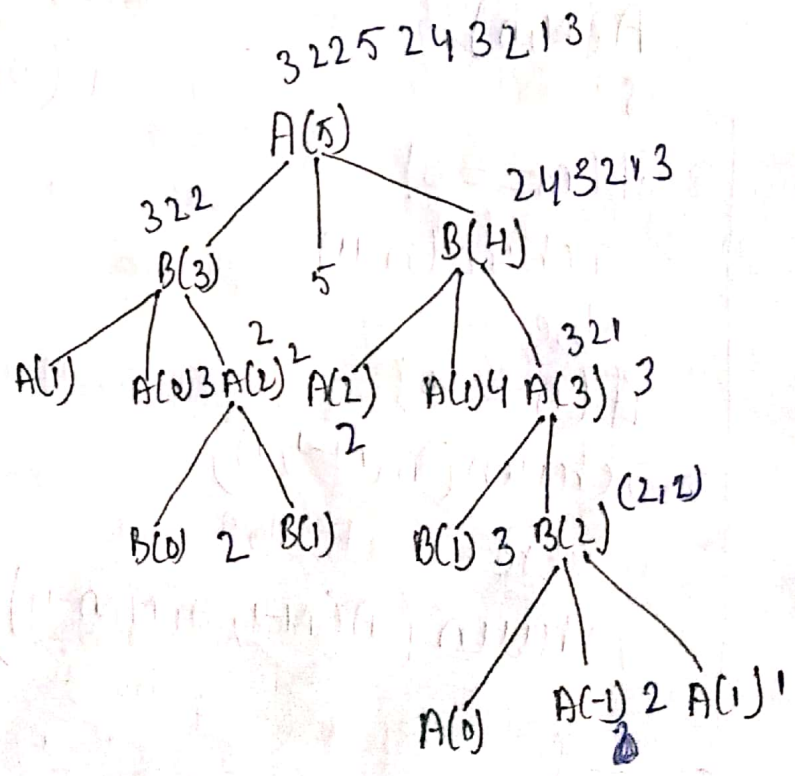
Recursion  $\Rightarrow$  A() calling A()  
 Indirect Recurs  $\Rightarrow$  A() calling B() & B() calling A()

```

A(n)
{
 if (n <= 1)
 return;
 else
 {
 B(n-2);
 Pf(n);
 B(n-1);
 }
}

B(n)
{
 if (n <= 1)
 return;
 else
 {
 A(n-2);
 A(n-3);
 Pf(n);
 A(n-1);
 Pf(n);
 }
}

```



i/p: A(5)

O/p: 3 2 2 5 2 4 3 2 1 3

With DP

dilemma

Time: A also n-people  
B also n-peoples  
Total Distinct calls

Space = 2n ⇒  $O(n)$

$4B + nB + 2nB$   
 $3n ⇒ O(n)$

without DP

Time: 1st function Program 2 function call  
2n & 2nd Program 3 function call  
Take upper bound

=  $O(3^n)$

Space: i/p + extra.  
 $4B + nB$

$O(n)$

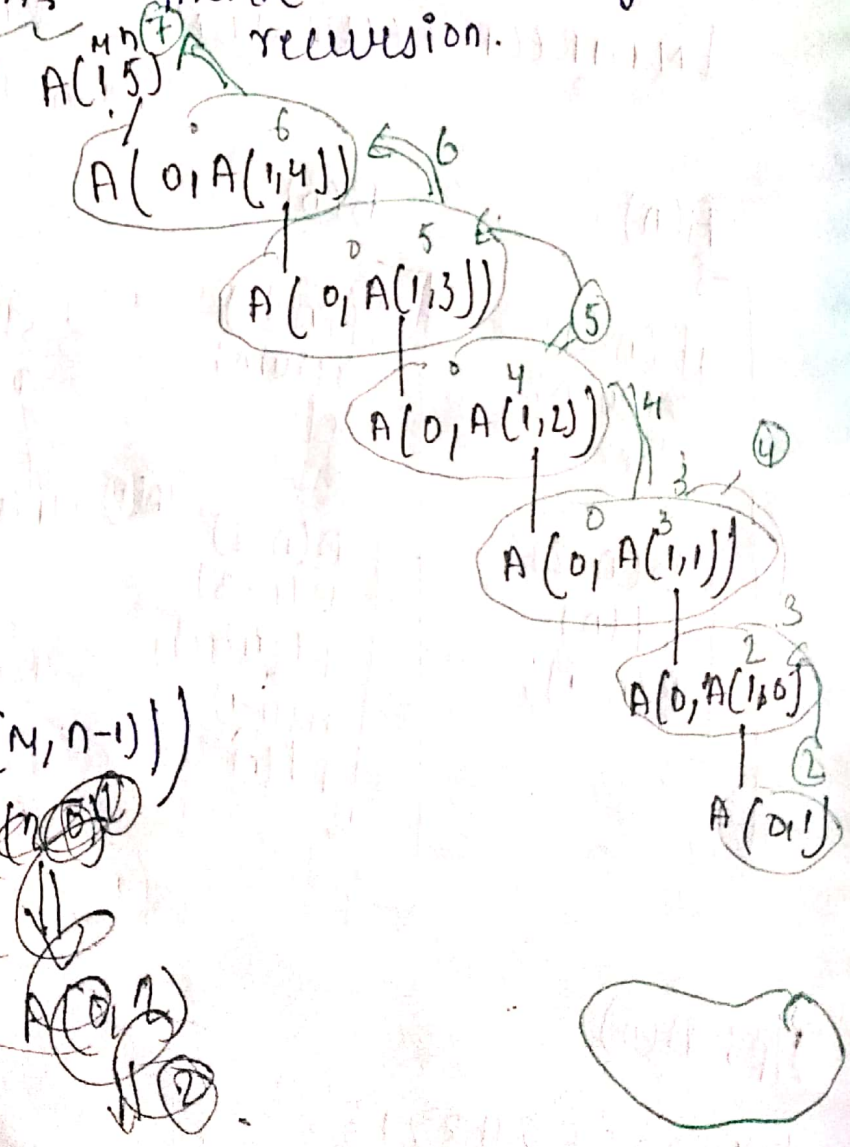
NESTED-RECURSION: Inside recursion again recursion.

```

A(m, n)
{
 if(m == 0)
 return(n+1)
 else
 if(n == 0)
 return(A(m-1, 1))
 else
 return(A(m-1, A(m, n-1)))
}

```

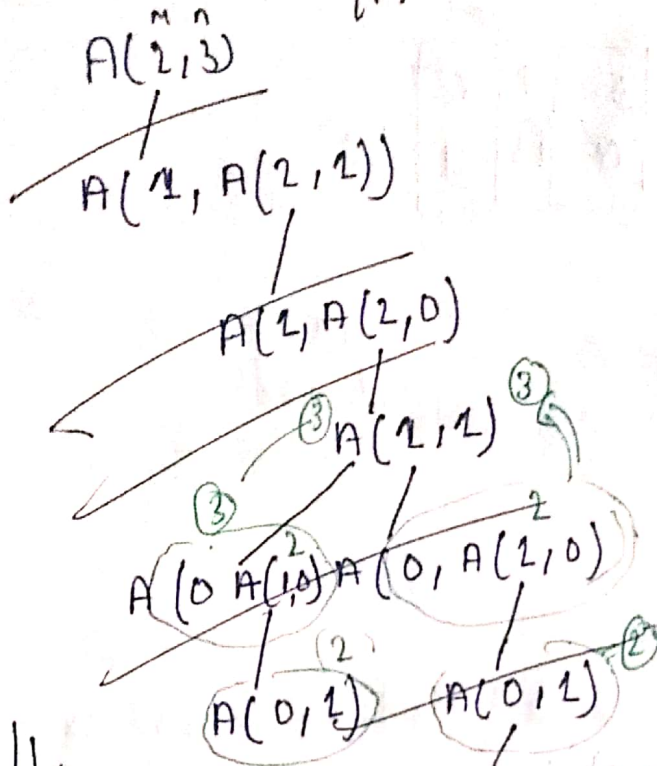
$A(1, 5)$



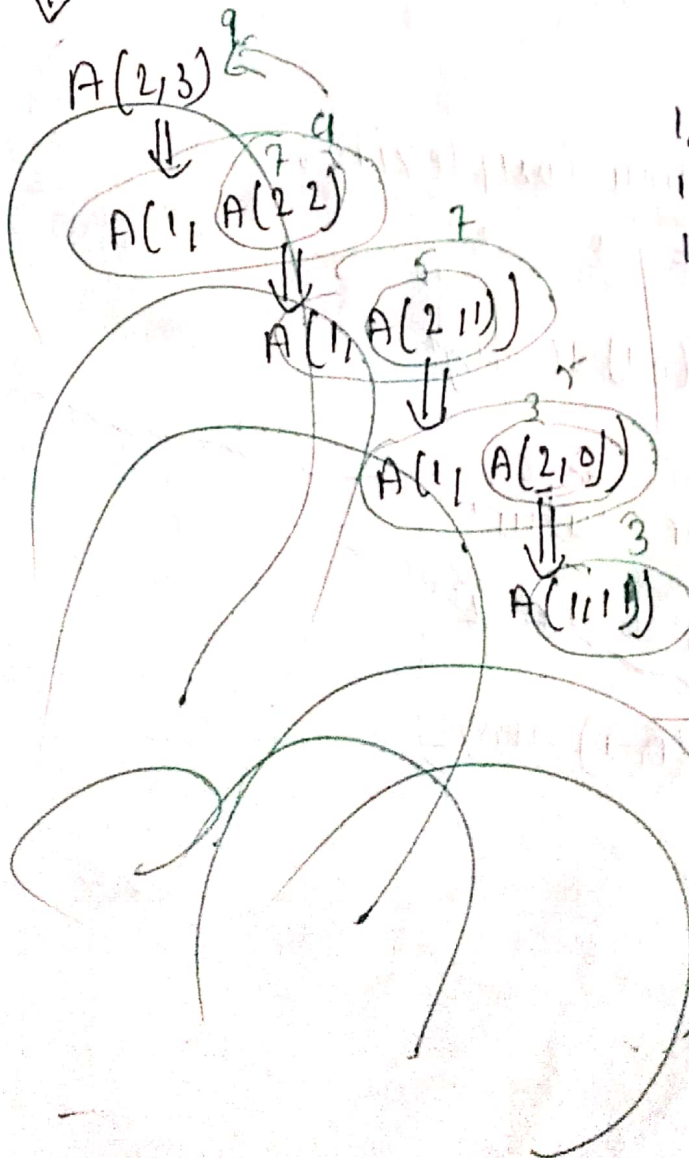
i/p: A(1, 5)

o/p: 7

1/P:  $A(2,3)$       0/P: 9



⇓



Observe for simplicity

|           |  |           |
|-----------|--|-----------|
| $1,0 = 2$ |  | $2,0 = 3$ |
| $1,1 = 3$ |  | $2,1 = 5$ |
| $1,2 = 4$ |  | $2,2 = 7$ |
| $\vdots$  |  | $2,3 = 9$ |
| $1,5 = 7$ |  |           |

## FIBONOLLI SERIES:

|      |   |   |   |   |   |   |   |    |    |
|------|---|---|---|---|---|---|---|----|----|
| n    | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7  | 8  |
| f(n) | 0 | 1 | 1 | 2 | 3 | 5 | 8 | 13 | 21 |

f(n)

{

if (n == 0 || n == 1)

return(n);

else

{

return(f(n-2) + f(n-1));

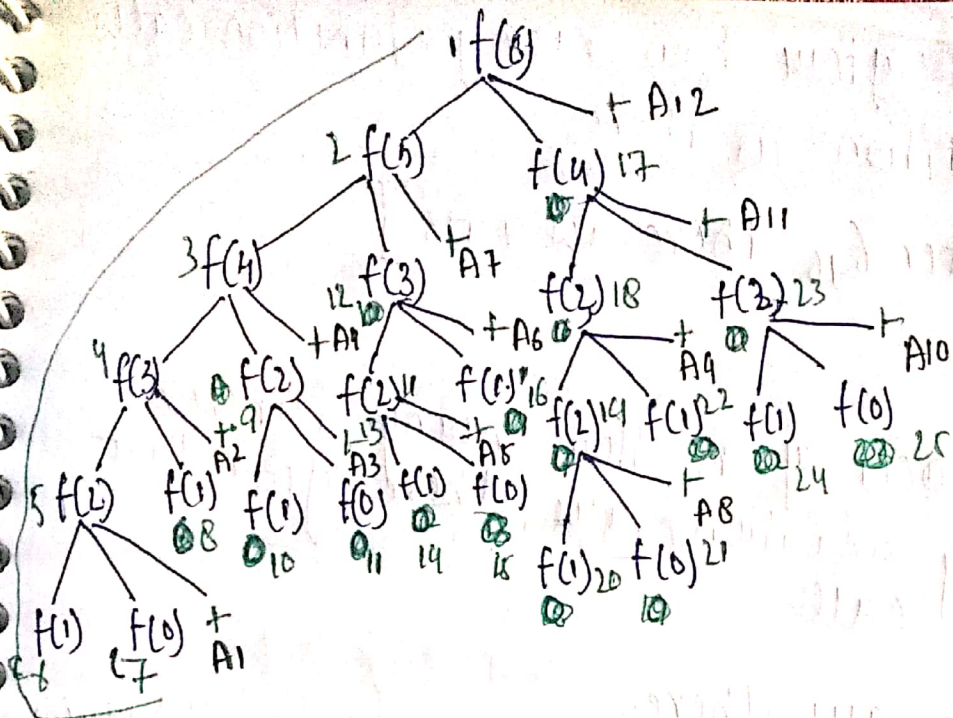
}

Recurrence Relation for Time Complexity:

$$T(n) = \begin{cases} O(1) & \text{if } n = 0 \text{ or } n = 1 \\ T(n-2) + T(n-1) + c & \text{if } n > 1 \end{cases}$$

Recurrence Relation for value:

$$T(n) = \begin{cases} n & \text{if } n = 0 \text{ or } n = 1 \\ T(n-2) + T(n-1) & \text{if } n > 1 \end{cases}$$



Ques 1: In fibonacci of n. After How many function calls first ~~2~~ Addition taken place.

Ans: After 7 function call, Addition taken. Go left most path.

⇒ [n+1] After (7) there is a A2

↳ Condition: if the program is like this

else

```

{
a = f(n)
a = f(n-1) ← left side will be n not n/2
b = f(n-2)
c = a+b
return(c)
}

```

If it is given like this. ⇒

```

else
{
a = f(n-2) ⇒ left side will be n/2
b = f(n-1)
! = [n/2 + 2]
}

```

Ques 2: In fib(6) after how many function calls.  
6<sup>th</sup> addition is there:

Ans: 16 [After 6<sub>16</sub> there is 9 A<sub>6</sub>]

Ques 3: In fibonacci of n How many function calls are there after last function call.

Ans: 3 Addition are there.

Last function call is f(10) [A<sub>10</sub>, A<sub>11</sub>, A<sub>12</sub>] Addition are there

After last function call while backtracking in every level one addition required.

Ans is  $n/2$  [cos left path  $n/2$  right path  $n/2$  it is a fibonacci series high side will be  $n/2$ ]

$$n/2 = 6/2 = \boxed{3}$$

Ques 4: In fibonacci of n How many Addition there.

No. of Additions

$$f(0) = 0$$

$$f(1) = 0$$

$$f(2) = 1$$

$$f(3) = 2$$

$$f(4) = 4$$

$$f(5) = 7$$

$$f(100) = f(99) + f(98) + 1$$

$$\boxed{f(n) = f(n-1) + f(n-2) + 1} \leftarrow \text{Rec. Rel}^n \text{ for Addition.}$$

ans: In fibonacci(n) how many function calls are there.

No of function calls

$$f(0) = 1$$

$$f(1) = 1$$

$$f(2) = 3$$

$$f(3) = 5$$

$$f(4) = 9$$

⋮

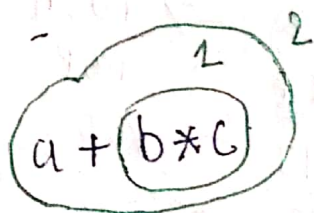
$$f(n) = f(n-1) + f(n-2) + 1$$

$$\boxed{f(n) = f(n-1) + f(n-2) + 1} \in \text{Rec. Rel}^n \text{ for function calls}$$

INFIX TO POSTFIX:

Ex:

Infix:



Postfix:

$$\frac{abc* +}{2}$$

Prefix:

$$\frac{+a*bc}{2}$$

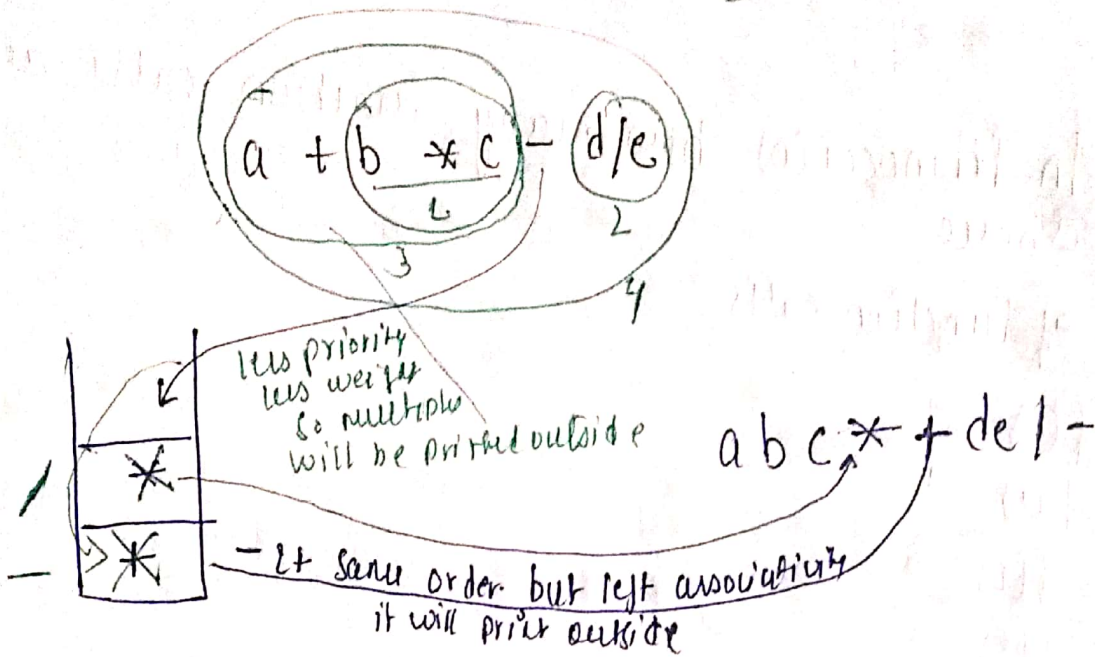
User like = Infix  
Processor = Postfix

NOTE: In all the above three notations operands order cannot change but operators order may change.

Ex 20

Infix:  $a + b * c - d / e$

If two operators having same priority then associativity will come. left to right. ( $*$  same,  $+$  same)



• Infix postfix conversion uses operator stack

Operand  $\Rightarrow$  Print

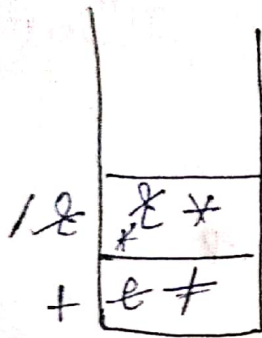
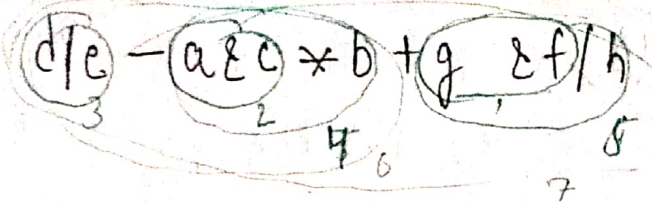
Operator

- ① TOP < NEXT  $\Rightarrow$  PUSH
- ② TOP > NEXT  $\Rightarrow$  POP
- ③ TOP  $\stackrel{\text{left to right}}{=}$  NEXT  $\Rightarrow$  POP
- ④ TOP  $\stackrel{\text{right to left}}{=}$  NEXT  $\Rightarrow$  PUSH

Prefix:  $- + a * b c / d e$

Diagram showing the prefix expression  $- + a * b c / d e$  with brackets indicating the order of operations: 1 for  $a * b$ , 2 for  $c / d$ , 3 for  $+ (a * b) c$ , and 4 for  $- + (a * b) c / d e$ .

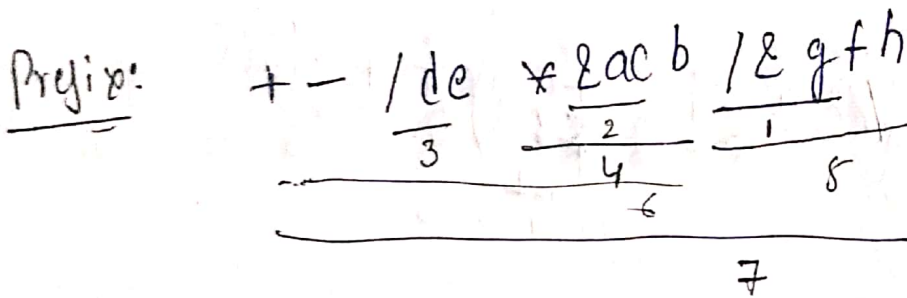
Ex 5:



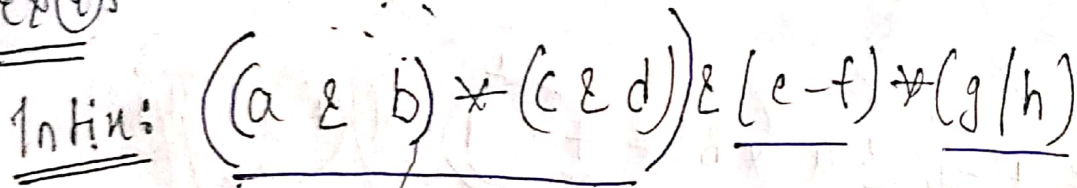
$- d/e - a \& c * b + g \& f / h +$

2-stack space.

Infix-Postfix conversion will take  $O(n)$  time

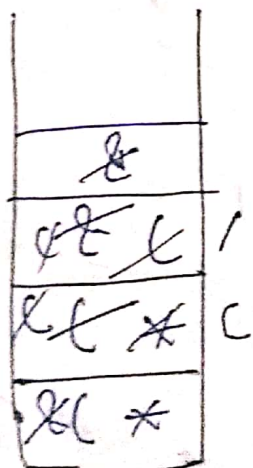


Ex 6:



Closed brackets  
comes  
it's pop & until open  
& print

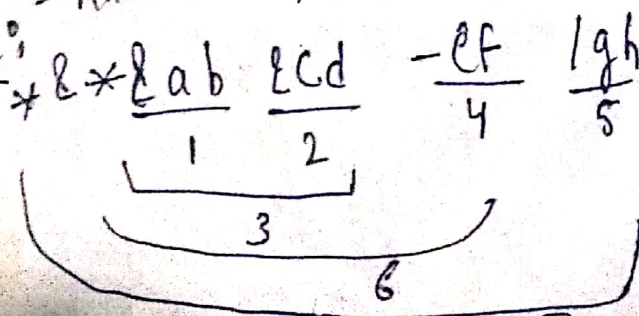
Post & Prefix having  
no brackets



Post fix:

$a b \& c d \& * e f - \& g h / *$

Prefix: - have no algorithm



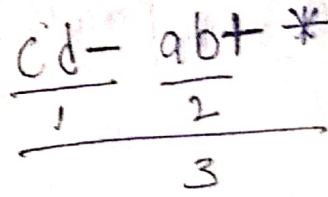
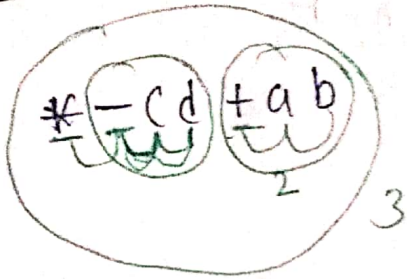
# Prefix to postfix

Ex 1%

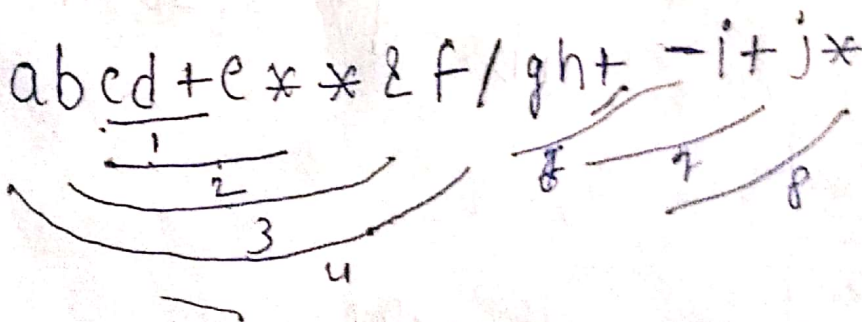
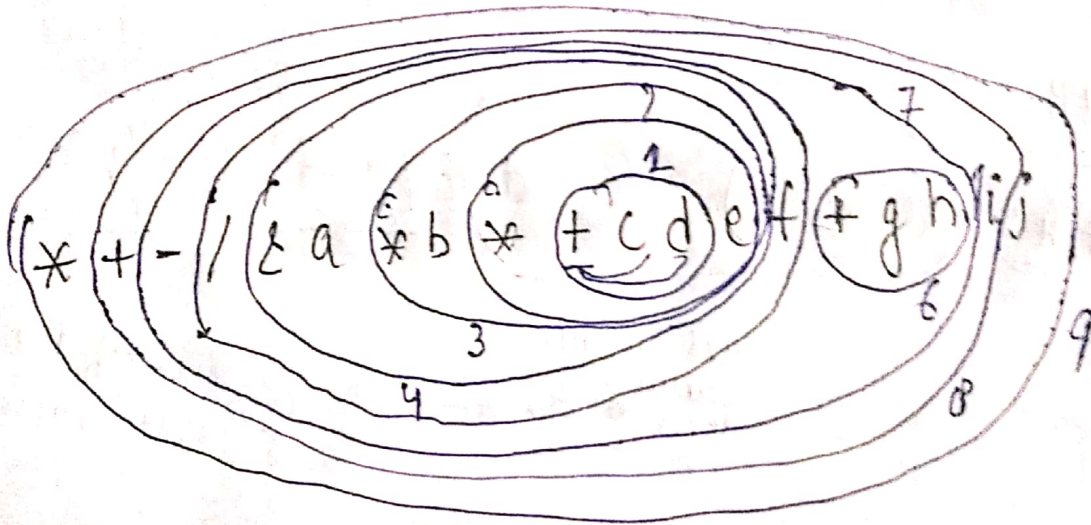
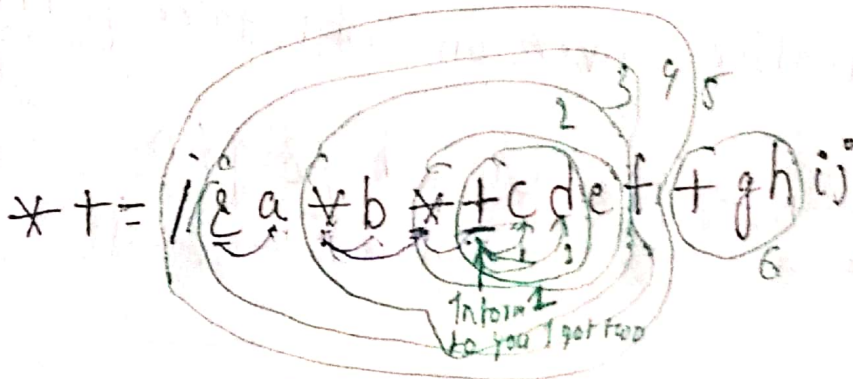
Prefix

Postfix

When you will round when 3 people there 2 operator 2 operand

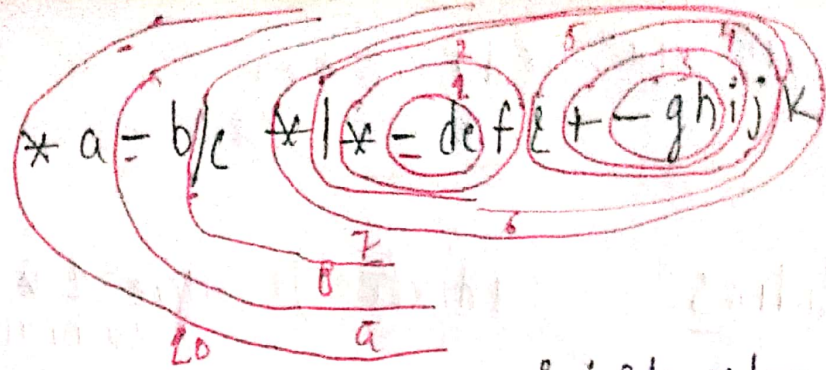


## Prefix:



Ex 3:

Prefix:



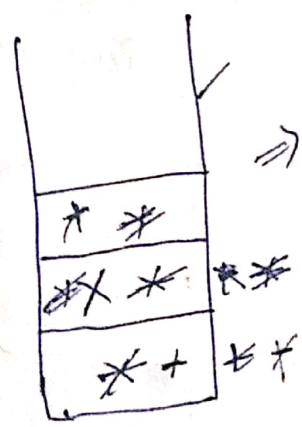
$$\frac{de-fx}{1} \quad \frac{\quad}{2}$$

$$\frac{gh-i+j \&/k *|-*}{3} \quad \frac{\quad}{4} \quad \frac{\quad}{5} \quad \frac{\quad}{6} \quad \frac{\quad}{7}$$

Postfix Evaluation:

Infix:  $2 \times 3 - 5 \times 3 / 5 \times 3 - 7 \times 2 + 6 \times 1 - 3$

Postfix: Infix to Postfix which will take  $O(n)$  time.



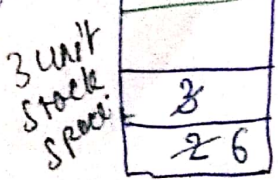
$$\Rightarrow 2 \ 3 \ * \ 5 \ 3 \ * \ 5 \ / \ 3 \ * \ - \ 7 \ 2 \ * \ - \ 6 \ 1 \ * \ + \ 3 \ -$$

$$2 \ 3 \ * \ 5 \ 3 \ * \ 5 \ / \ 3 \ * \ - \ 7 \ 2 \ * \ - \ 6 \ 1 \ * \ + \ 3 \ -$$

↓ Postfix Evaluation.  
In this algorithm operators are pushed.

If you saw Operator stop. two time pop & perform that op<sup>n</sup>

Operand Stack.

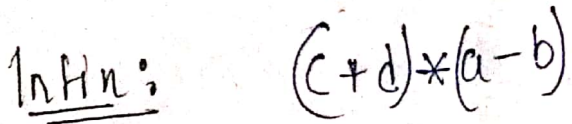
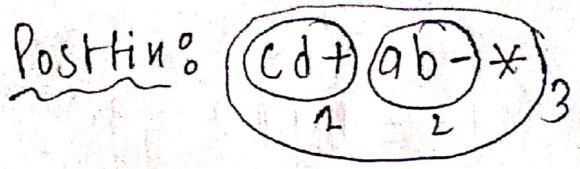


```

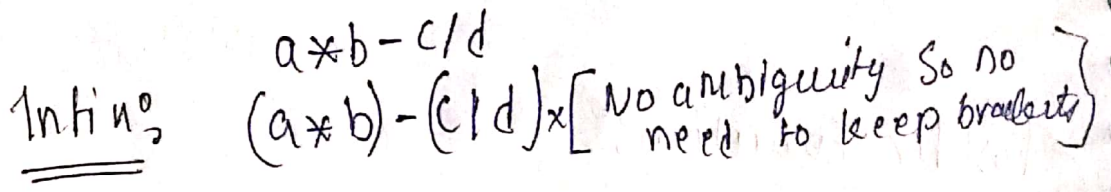
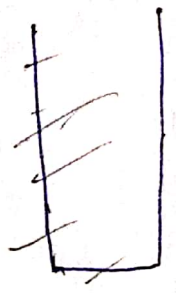
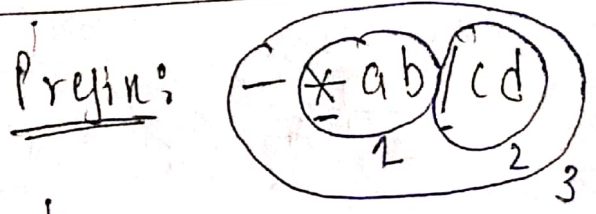
if (operand)
 push
if (operator)
 &
 Operator2 = POP()
 Operator1 = POP()
 Y = OP1 OP2
 push(Y)

```

Problem:



Here 2 is dxa  
so brackets may be  
required



NOTE:

To evaluate postfix expression one scan is enough. but to evaluate infix & prefix more than 2 scan required.

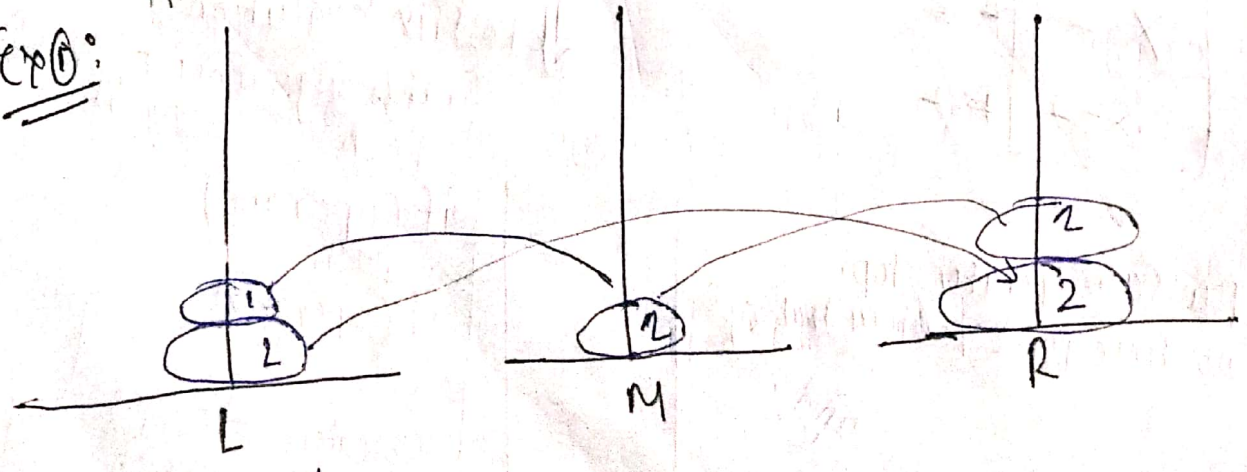
21/Sept

Towers of Hanoi:

Penuli removed  
work penuliy.

Exp:

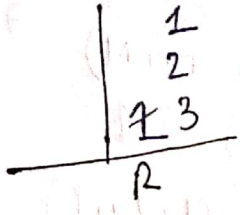
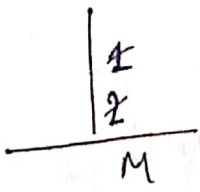
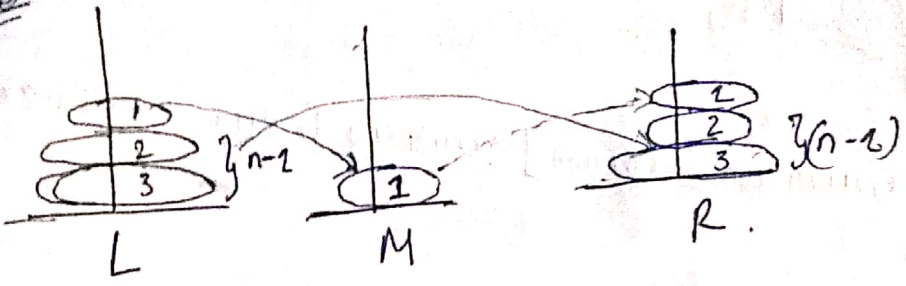
$n=3$



- L-M
- L-R
- M-R

$n=3$

Ex 1

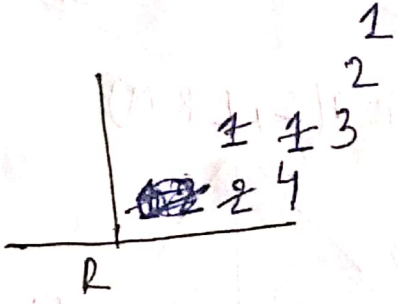
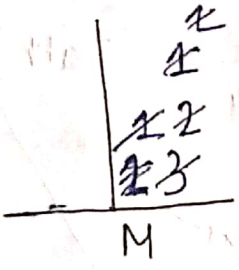
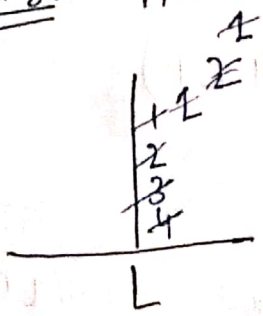


L-R  
L-M  
R-M

L-R | M-L  
M-R  
L-R

Ex 2

$n=4$



L-M  
L-R  
M-R  
L-M  
R-L  
R-M  
L-M

L-R | M-R  
M-L  
R-L  
M-R  
L-M  
L-R  
M-R

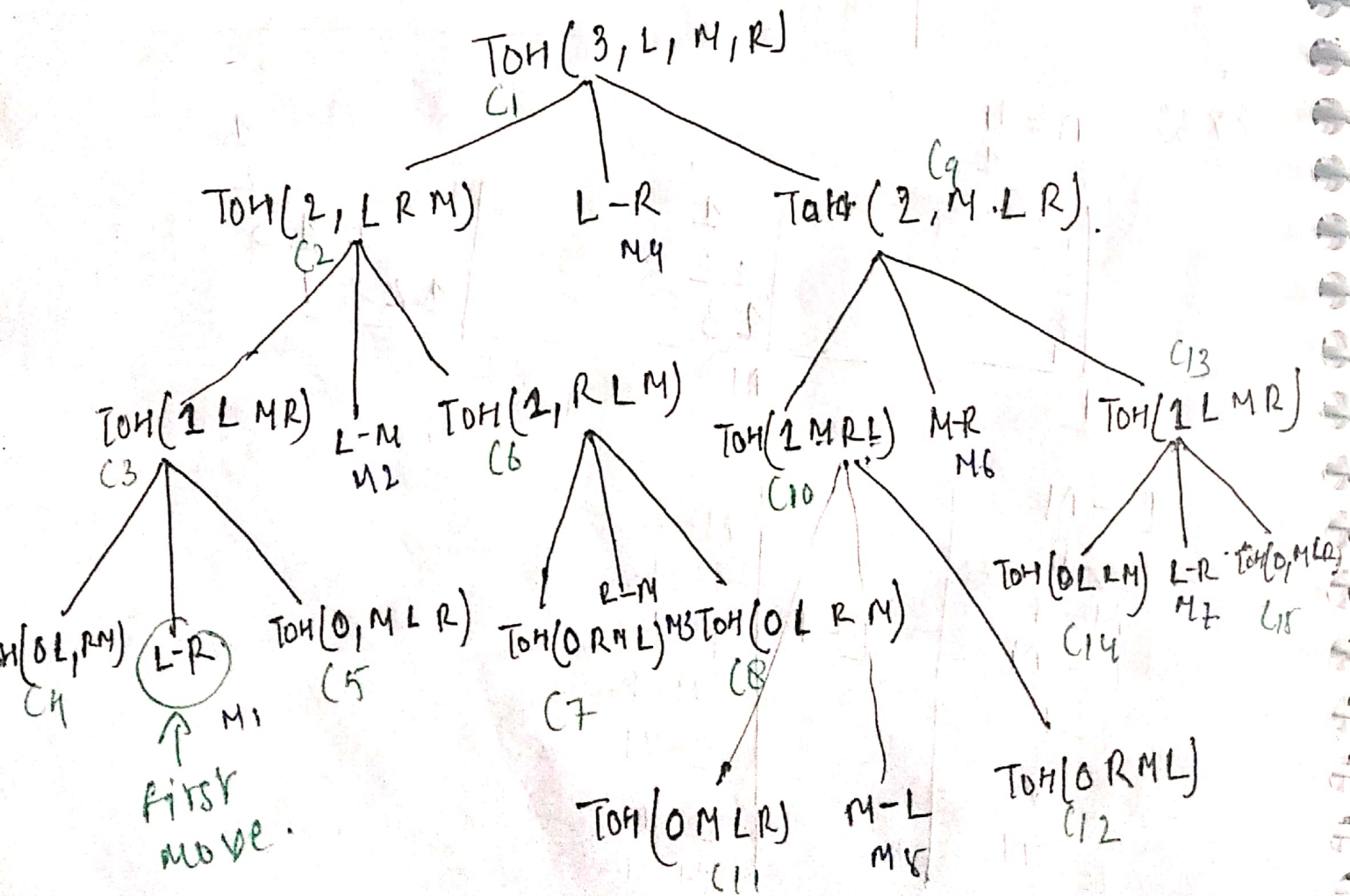
# Algorithm:

$T(n, L, M, R) \Rightarrow T(n)$

```

{
 if (n == 0) [termination condition] sometimes in ques. it will be 2 also
 returns;
 else
 {
 T(n-1, L, R, M) $\Rightarrow T(n-1)$
 MOVE (L-R)
 T(n-1, M, L, R) $\Rightarrow T(n-1)$
 }
}

```



Ques 1: In ToH of n After how many function call. first move will be take place. = (4)

Ans: Go to extreme left.  $[n+1]$  [If the termination condition is 0]  
After  $C_4$  there is a  $M_1$ .  
If its termination condition is 1 then Ans is  $[n] = 3$

Ques 2: In ToH(3) after how many function call there is a move from middle to right.

Ans: 12  
L-R  
L-M  
R-M  
L-R  
M-L  
M-R  
After  $C_{12}$  there is a  $M_6$

Ques 3:  $\therefore$  In ToH of n ToH(n) Is any move after last function call.

Ans: No work in the balltrailing, so No move after last function call.  
After  $C_{15}$  in Balltrailing No work.

Ques: In ToH(n) is there is any function call after last move:

Ans: 1 function,  $\therefore C_{15}$

ans: In  $T(n)$  How many moves there?

Ans:

$$T(0) = 0 \text{ move}$$

$$T(1) = 1 \text{ move}$$

$$T(2) = 3 \text{ move}$$

$$T(3) = 7$$

$$T(4) = 15$$

$$T(n) = 2T(n-1) + 1$$

$$T(n) = 2T(n-1) + 1$$

$$= \boxed{2^n - 1}$$

ans: In  $T(n)$  how many function calls are there

$$T(0) = 1$$

$$T(1) = 3$$

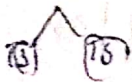
$$T(2) = 7$$

$$T(3) = 15$$

$$T(n) = 2T(n-1) + 1$$

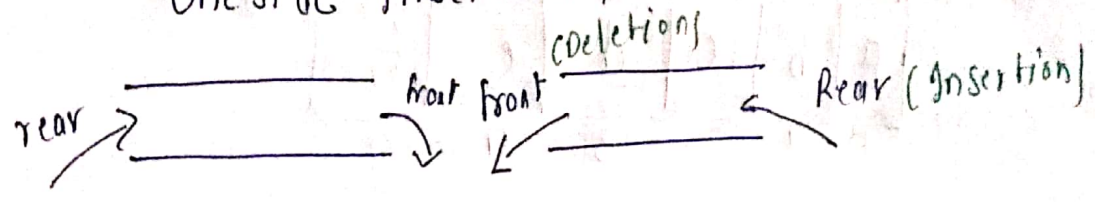
$$T(n) = 2T(n-1) + 1$$

$$= \boxed{2^{n+1} - 1}$$



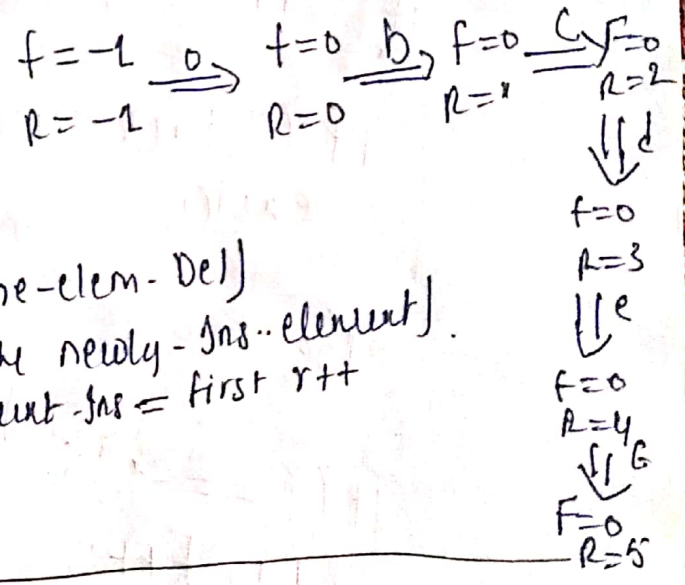
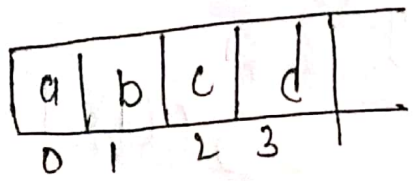
# Queue

Defination: Both the Side open. but the condition is one side Insertion, other side deletion



Insertion will be done w.r.t. 'Rear'  
 Deletion " " " " "Front"

- \* Front: Front is a variable which contain. position of the element. to be deleted
- \* Rear: Rear is a variable which contain. position of the newly inserted element.



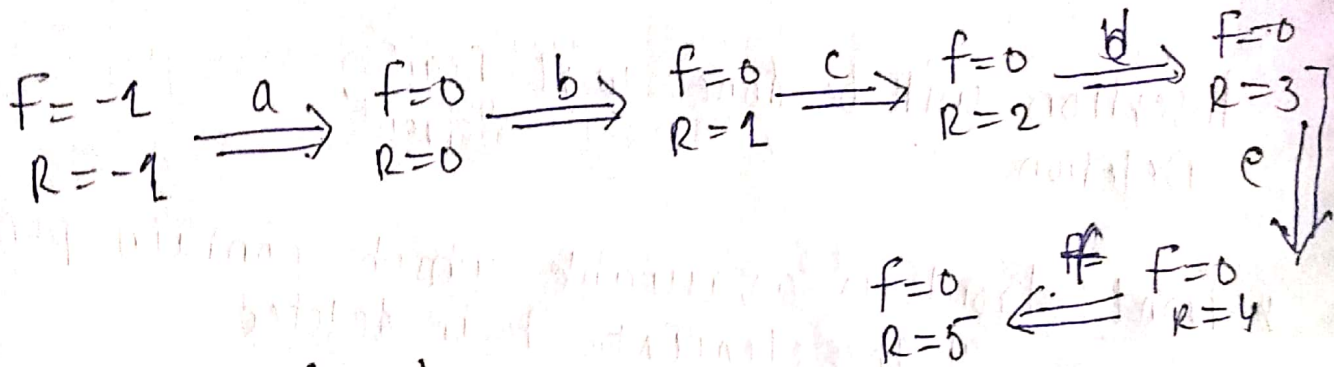
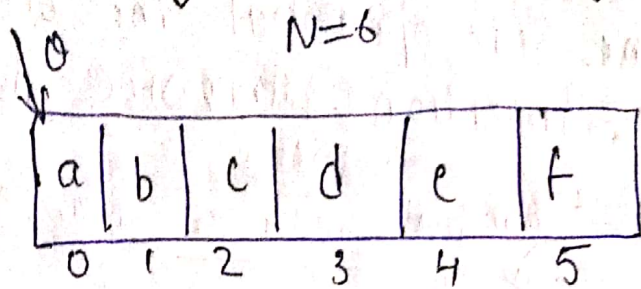
Front: 0 (Position of the elem. Del)  
 Rear: 3 (Position of the newly-ins. element).  
 Next-Element-Ins = first r++

Property: FIFO or LIFO

AOT of Queue

- ⇓
- ① Enqueue()
  - ② Dequeue()

Ques: Implementing Queue Using array:



Void Enqueue(int x)

{  
if  $(R + 1 == N)$  or  $R = N - 1$

{  
  Print ("Stack Overflow");  
  exit(1);  
}

⇒ O(1) [EC]

else.

{  
  1<sup>st</sup> Insertion

if  $(R == -1)$   
   $R++, F++;$

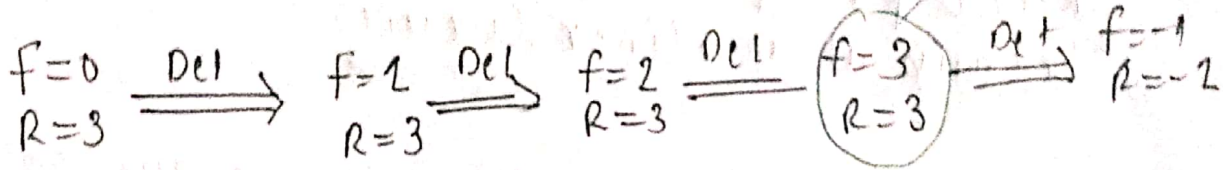
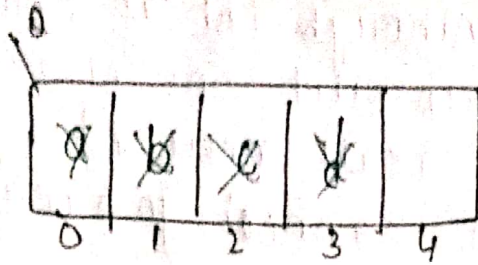
else

$R = R + 1$

$O[R] = x$

}

}



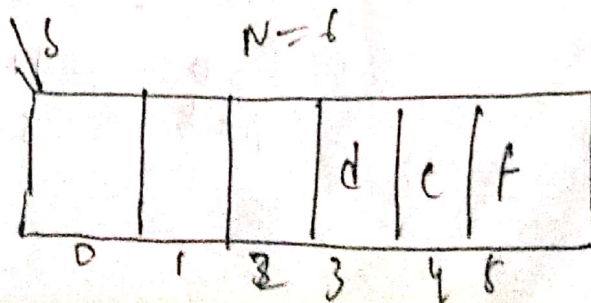
Dequeue()

```

{
 if (F == -1)
 printf("Queue is underflow");
 exit(1);
 else
 {
 y = arr[F] // first store the 2nd element
 if (F == R) // Deleting last element then
 F = R = -1;
 else
 F = F + 1;
 }
}

```

Using above program we can implement Queue. but it is not efficient.

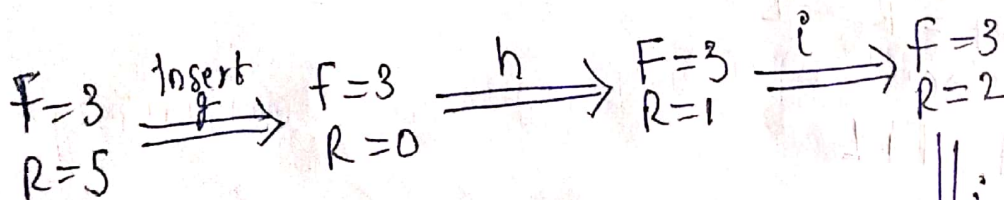
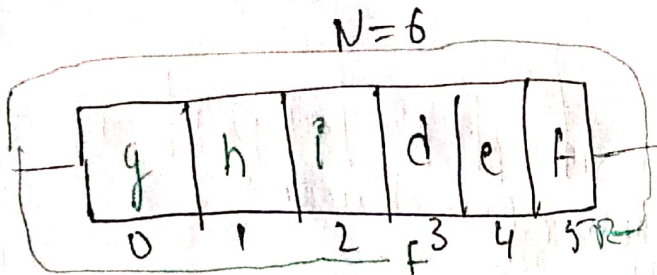


F=3  
R=5

In the above queue even though lot of space available we cannot insert one more element further, coz rear reached right most part place & it cannot go back because it is a linear queue.

To Implement Queue efficiently we are going to the Circular Queue.

### Circular Queue



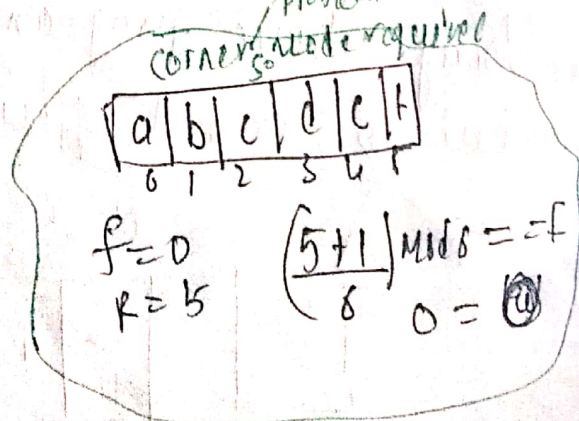
(corner problem happen so mod required)

```

void Enqueue(int n)
{
 // change
 if((R+1)%N == F)
 {
 printf("Queue Overflow");
 exit(1);
 }
 else
 {
 if(R == -1)
 R++, F++;
 }
}

```

$$\text{if } (R+1) \% N == F$$
 (corner problem)

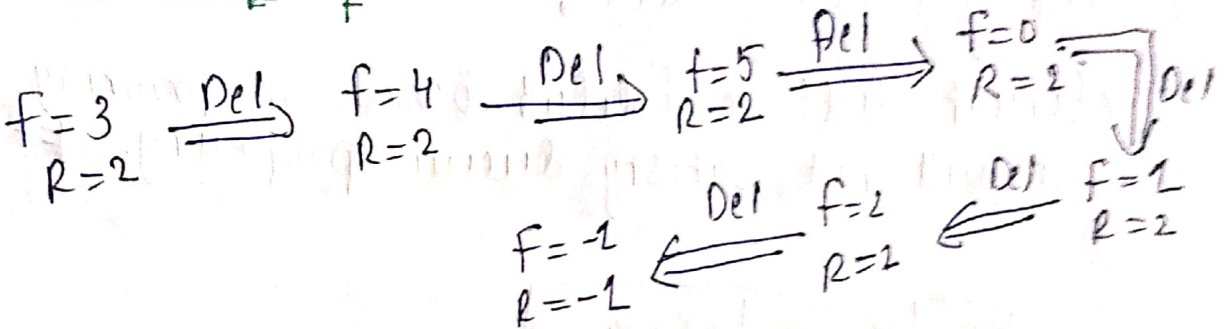
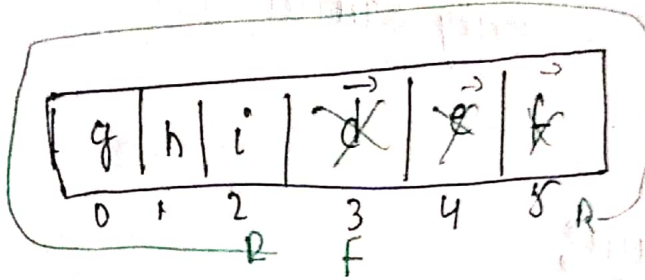


else

$$R = (R + 1) \text{Mod } N \quad \text{Change}$$

$$O[R] = k$$

}  
}



```
int Dequeue()
```

```
{
 if (F == -1)
```

```
{
 printf("Queue is underflow");
 exit(1)
```

```
}
else
```

```
{
 y = O[F]
```

```
 if (F == R)
```

```
 F = R = -1
```

```
 else
```

```
 F = (F + 1) Mod N only change
 return(y);
```

```
}
}
```

# Priority Queue:

Ascending  
Priority Queue

Descending  
Priority Queue

## Ascending Priority Queue:

Element will be deleted based on priority.  
It will not satisfy Queue property. ~~FIFO~~.

|               |    |               |    |               |    |    |
|---------------|----|---------------|----|---------------|----|----|
| <del>25</del> | 80 | <del>17</del> | 99 | <del>36</del> | 15 | 28 |
|---------------|----|---------------|----|---------------|----|----|

15 25 26 . . . . . (whoever the highest priority should be deleted first)

## Implementing Ascending Priority Queue: 3 way to represent

① Using unsorted array.

25, 80, 99, 17, 36, 15, 28

|   |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|
| a | 25 | 80 | 99 | 17 | 36 | 15 | 18 |
|   | 0  | 1  | 2  | 3  | 4  | 5  | 6  |

$i = 0$   
 $j = 6$

Enqueue:

ist = 0  
 $i++$   
 $j++$   
 $a[i] = n$

Remaining  
 $j++$   
 $a[j] = n$

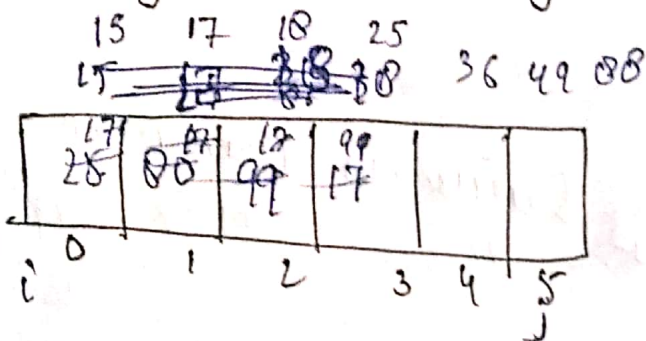
} O(1)

DeDeDe:

①  $k = \text{position min element} \Rightarrow O(n)$  } (computing & scan to find minimum element)

②  $y = a[k]$   
 $a[k] = a[j]$  }  $O(n)$   
 $j = j - 1$   
return(y) }  $O(n)$

② Using Sorted Array:



EO:

- ① Store n in j<sup>th</sup> place. }  $O(n)$
- ② keep n in correct place }  $O(n)$

1-EO  
 ↓  
 O(n)

PO:

```

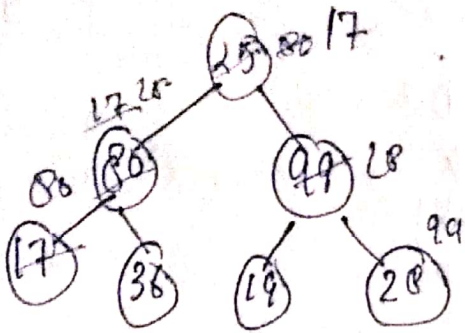
y = a[i]
i++
return(y)

```

1-PO  
 ↓  
 O(n)

③ Using Min heap Best way

25, 80, 99, 17, 36, 19, 28.

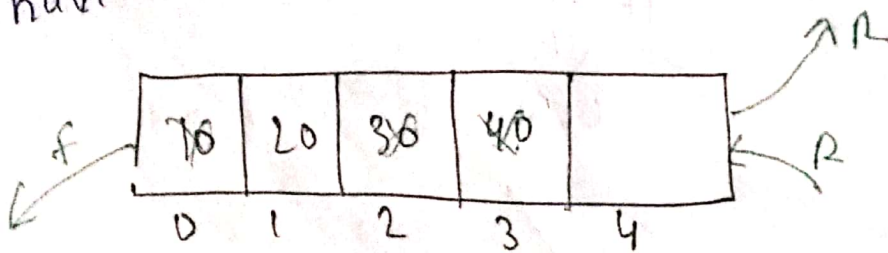


$1 - EO \Rightarrow \log n$   
 $1 - DO \Rightarrow \log n$   
 $\underline{O(\log n)}$

Ascending priority: Queue is min heap  
 Descending priority: " " max heap.

Input-Restricted Queue?

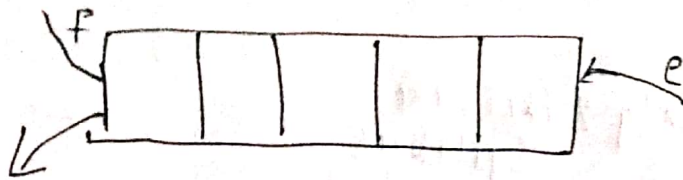
Both the side we can do deletion. only Input have restriction.



$F = 0 \ 2$   
 $R = 2 \ 3 \ 4 \ 3 \ 2$

$\frac{EO}{R++}$        $\frac{DO-F}{FH}$        $\frac{DO-R}{R--}$       Not FIFO

## Output restricted queue:



$$\frac{DQ}{F++}$$

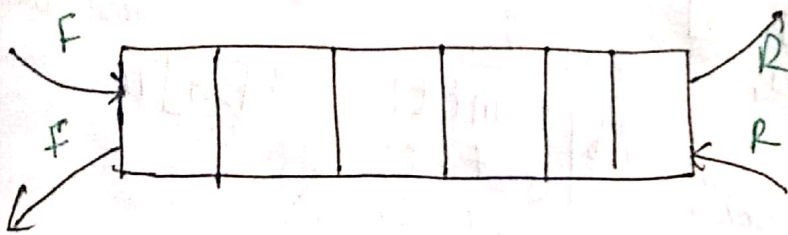
$$\frac{EQ - R}{R++}$$

not-FIFO

$$\frac{EQ - F}{F--}$$

## Double-Ended Queue: (Deque)

L Depending upon context you have to consider



$$\frac{EQ - R}{R++}$$

$$\frac{DQ - F}{F++}$$

$$\frac{EQ - F}{F--}$$

$$\frac{DQ - R}{R--}$$

Deque Op<sup>n</sup>: Delete Element  
 Deque Data Structure: Double  
 Ended Queue

# Parameter Passing Techniques:

- ① Call By Value.
  - ② Call By Reference
  - ③ Call By Name
  - ④ Call By Need
  - ⑤ Call By Text
  - ⑥ Call By Copy-restore.
- } Only 2 are in the syllabus.
- } Not there in the syllabus.

ex ①: ~~Value~~ Call by Value

```

main()
{
 ① int a = 10, b = 20;
 ② Pf(a,b);
 ③ Swap(a,b);
 ④ Pf(a,b);
}

Swap(int c, int d)
{
 ① int t;
 ② t = c;
 ③ c = d;
 ④ d = t;
}

```

Diagram illustrating memory addresses and values:

- main() scope:**
  - Variable **a** at address 1000 contains value 10.
  - Variable **b** at address 2000 contains value 20.
- Swap(int c, int d) scope:**
  - Parameter **c** at address 3000 contains value 20 (copied from b).
  - Parameter **d** at address 4000 contains value 10 (copied from a).
  - Local variable **t** at address 5000 contains value 10.

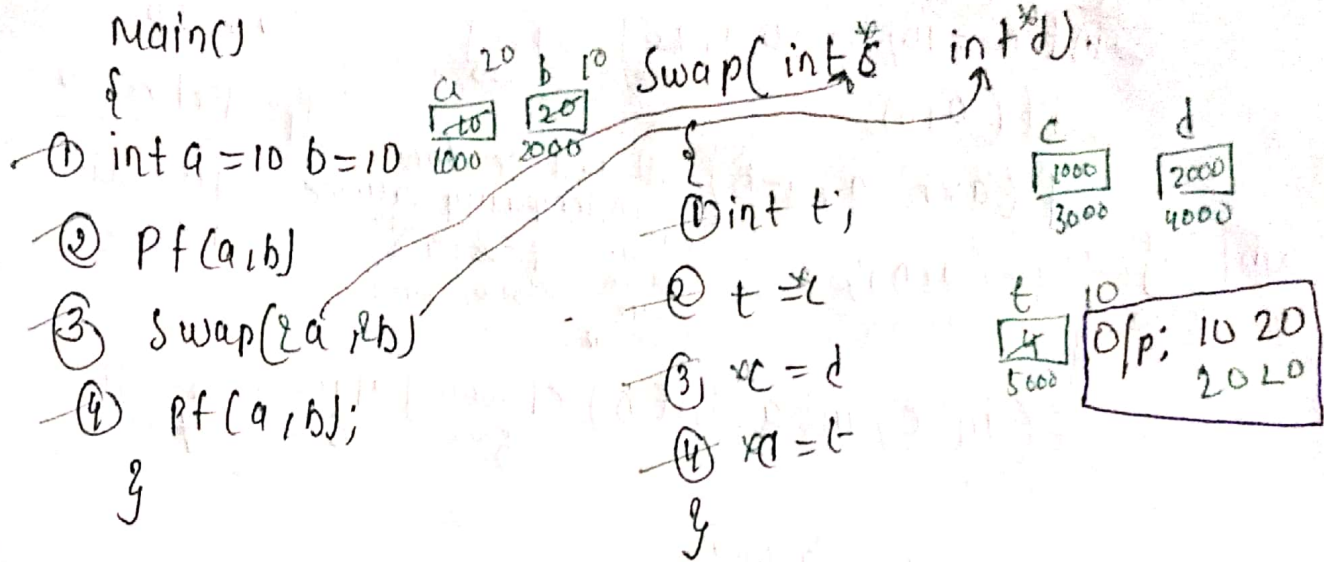
Annotations:

- "copy from value at b" with an arrow pointing from b to c.
- "copy from a" with an arrow pointing from a to d.
- "Par-Pas. Tech active here." pointing to the Swap function call.
- "original all not affected." pointing to the main function scope.

O/p: 20, 20  
10, 20

- ① In Call by Value Original will not affect because modification is done in copy.
- ② If the No's of parameters are very less then go to call by value.

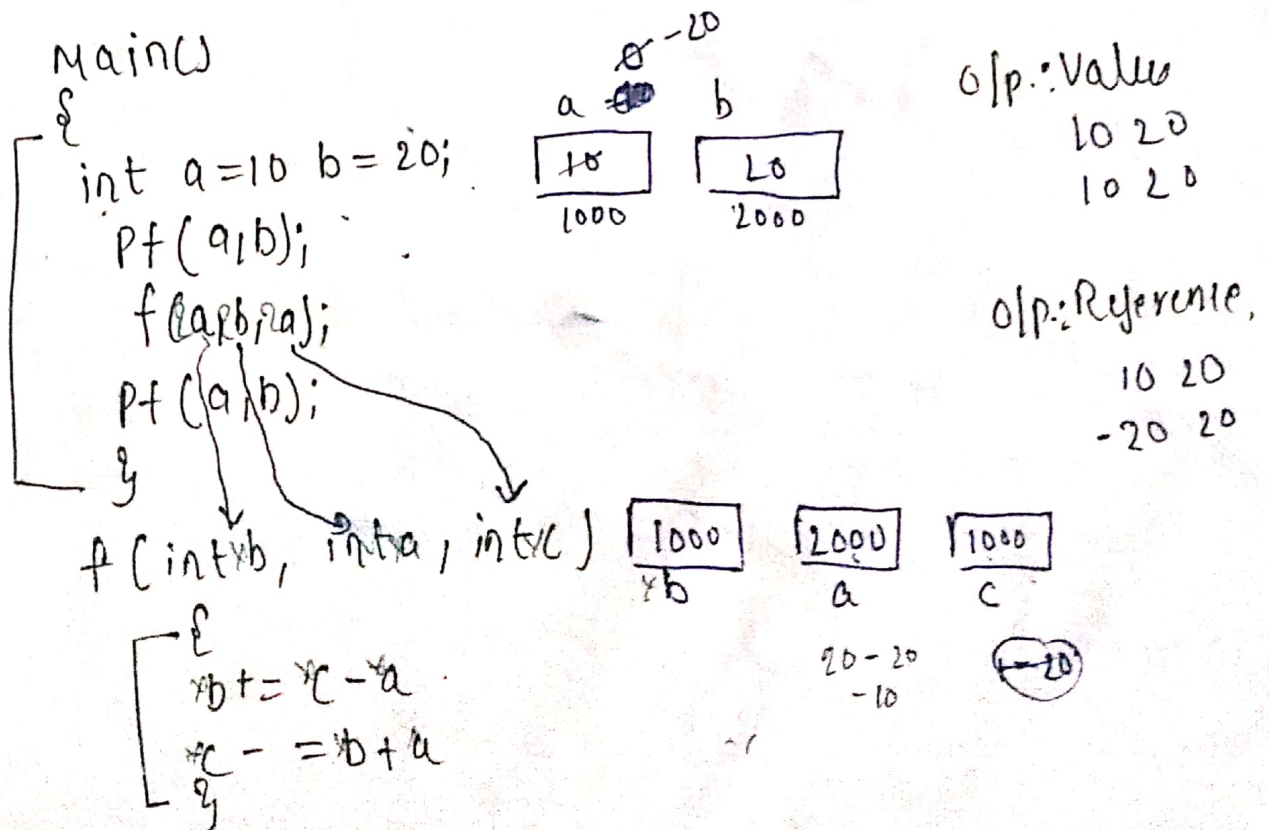
② Call by Reference?  
 ↳ Compiler will do.



① In call by reference original will be affected because modification with same w.r.t address.

② If you want pass more parameter call by reference is better option.

Ques: Consider the following C program.



Ex 5: main()

Call by ~~ref~~ Reference.

O/p: Value

```
{
 int a = 10, b = 20
 a [20] 1000
 b [20] 2000
```

10 20  
10 20

```
 pf(a, b);
```

O/p: Reference

```
 f(a * b, a - b)
```

It will be done in temporary variable

10 20  
100 20

```
 pf(a, b);
}
```

That place address will be set

[10] a-b  
-3000 Address

```
 f(int *c, int a, int b)
 c [4000] 3000
 a [1000] 8000
 b [3000] 7000
```

```
 *c = *a + 10;
```

```
 *b = *a + c
```

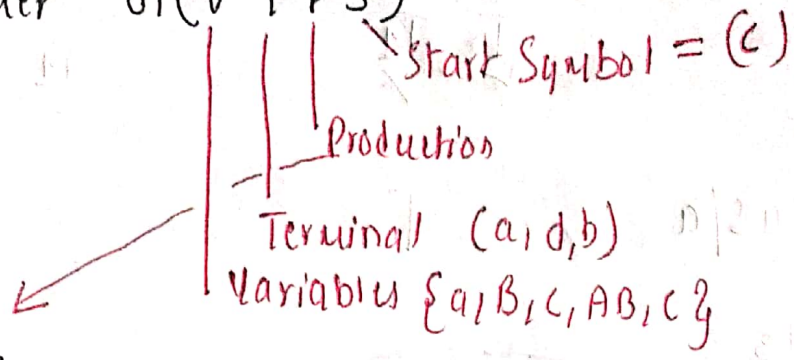
```
 *a = 10
```

```
}
```

8/Oct/2019

# COMPILER DESIGN

Grammar  $G(V, T, P, S)$



- $S \rightarrow ABC$
- $A \rightarrow BC$
- $B \rightarrow ac$
- $C \rightarrow d|b$

## Types of Grammar:

According to Chomsky

- ① Type-0 [Unrestricted Grammar]
- ② Type-1 [Context-free Grammar]
- ③ Type-2 [Context-sensitive Grammar]
- ④ Type-3 [Regular Grammar]

### ① Type 0:

$$\alpha \rightarrow \beta$$

$$\alpha, \beta \in (V+T)^*$$

### ② Type 1:

- ① It should be type 0
- ②  $|\alpha| \leq |\beta|$

### Type 2:

- ① Type 1
  - ②  $A \rightarrow B$
- A is a single variable.  
 $\beta \in (V+T)^*$

### Type 3:

- ① Type 2
  - ②  $A \rightarrow BT^*|T^*$
- (Either) left linear (or) right linear
- ②  $A \rightarrow TB|T^*$
- Right-linear
- Both cannot be there  
 $A, B$  are variables

two variables cannot be side by side  
 $S \rightarrow ABCA$